Logic for Computer Science

Seminar 3: Relations and Functions

October 19, 2016

1. Why is f not a function from $\mathbb{R}$ to $\mathbb{R}$ if
   (a) $f(x) = 1/x$?
   (b) $f(x) = \sqrt{x}$?
   (c) $f(x) = \pm\sqrt{x^2+1}$?
2. Determine whether f is a function from $\mathbb{Z}$ to $\mathbb{R}$ id
   (a) $f(n) = \pm n$.
   (b) $f(n) = \sqrt{n^2+1}$.
   (c) $f(n) = \frac{1}{n^2-4}$.
3. Find these values:

a) $\lfloor 1.1 \rfloor$	b) $\lceil 1.1 \rceil$	c) $\lfloor -0.1 \rfloor$
d) $\lceil -0.1 \rceil$	e) $\lceil 2.99 \rceil$	f) $\lceil -2.99 \rceil$
g) $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor$	h) $\lceil \lfloor \frac{1}{2} \rfloor + \lceil \frac{1}{2} \rceil + \frac{1}{2} \rceil$	

4. Determine whether each of these functions from $\{a, b, c, d\}$ to $\{a, b, c, d\}$ is injective:
   (a) $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$
   (b) $f(a) = b$, $f(b) = b$, $f(c) = d$, $f(d) = c$
   (c) $f(a) = d$, $f(b) = b$, $f(c) = c$, $f(d) = d$
5. Determine whether each of these functions from $\mathbb{Z}$ to $\mathbb{Z}$ is injective:
   (a) $f(n) = n - 1$,
   (b) $f(n) = n^3$,
   (c) $f(n) = n^2 + 1$,
   (d) $f(n) = \lceil \frac{n}{2} \rceil$
6. Which of the following functions $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is injective?

(a) $f(m,n) = m+n$,
(b) $f(m,n) = m^2+n^2$,
(c) $f(m,n) = m$,
(d) $f(m,n) = |n|$,
(e) $f(m,n) = m-n$.

7. Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ that is
   (a) Injective but not surjective
   (b) Surjective but not injective
   (c) Neither injective nor surjective
8. Give an explicit formula for a function from $\mathbb{Z}$ to $\mathbb{N}$ that is
   (a) injective but not surjective.
   (b) surjective but not injective.
   (c) both injective and surjective.
   (d) neither injective nor surjective.
9. Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$:
   (a) $f(x) = -3x+4$
   (b) $f(x) = -3x^2+7$
   (c) $f(x) = (x+1)/(x+2)$
   (d) $f(x) = x^5+1$
10. Let $S = \{-2,-1,0,1,2,3\}$. Find $f(S)$ if
    (a) $f(x) = 1$
    (b) $f(x) = 2\,x+1$
    (c) $f(x) = \lceil x/5 \rceil$
    (d) $f(x) = \lfloor (x^2+1)/3 \rfloor$
11. Let $f(x) = 2\,x$. What is

    a) $f(\mathbb{Z})$? b) $f(\mathbb{N})$? c) $f(\mathbb{N})$?

12. Let f be a function from the set A to the set B. Let S and T be subsets of A.Show that
    (a) $f(S \cup T) = f(S) \cup f(T)$.
    (b) $f(S \cap T) \subseteq f(S) \cap f(T)$.
13. Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2+1$ and $g : \mathbb{R} \to \mathbb{R}$, $g(x) = x+2$. Find $f \circ g$ and $g \circ f$.
    Remember that, if $f : B \to C$ and $g : A \to B$ then $f \circ g : A \to C$ and $(f \circ g)(x) = f(g(x))$ for all $x \in A$.