Logic Programming

Lists. Recursion

- Lists
- Recursion
- Accummulators

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  5. change "do" to "no",
  6. leave everything else unchanged.

Recursive mapping (continued)

- The program:

```
change(you, i).
change(are, [am, not]).
change(french, german).
change(do, no).
change(X, X).

alter([],[]).
alter([H|T], [X|Y]):-
        change(H, X),
        alter(T, Y).
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- Note that this program is limited:
  - it would change “i do like you” into “i no like i”,
  - new rules would have to be added to the program to deal with such situations.

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- Use the predicate name/2 which returns the name of a symbol:

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X=alp.
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```
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X=alp.
```

- The program:

```
aless(X, Y):-
    name(X, L), name(Y, M), alessx(L,M).
alessx([], [_|_]).
alessx([X|_], [Y|_]):- X < Y.
alessx([H|X], [H|Y]):- aless(X, Y).
```

Homework exercises for today. Questions?

- Define predicates in Prolog for:
  1. The length of a list
  2. The sum of elements of a list
  3. The reverse of a list
  4. The list of elements on even positions
  5. The concatenation of two lists.

Append

- We want to append two lists, i.e.

```
?- appendLists([a,b,c], [3,2,1], [a,b,c,3,2,1]).
   true
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This illustrate the use of appendLists/3 for testing that a list is the result of appending two other lists.

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- Other uses of appendLists/3:

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- Other uses of appendLists/3:

- Total list computation:

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?- appendLists([a, b, c], [3, 2, 1], X).
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?- appendLists([a, b, c], [3, 2, 1], X).
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- Isolate:

```
?- appendLists(X, [2, 1], [a, b, c, 2, 1]).
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?-appendLists([a,b,c], [3,2,1], [a,b,c,3,2,1]).
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- Total list computation:

```
?-appendLists([a, b, c], [3, 2, 1], X).
```

- Isolate:

```
?-appendLists(X, [2, 1], [a, b, c, 2, 1]).
```

- Split:

```
?-appendLists(X, Y, [a, b, c, 3, 2, 1]).
```

```
        % the boundary condition
appendLists([ ], L, L).
        % recursion
appendLists([X|L1], L2, [X| L3]):-
    appendLists(L1, L2, L3).
```

Summary

- The recursive nature of structures (and in particular lists) gives a way to traverse them by recursive decomposition.

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- This process can be made more efficient: introduce an extra variable in which the "result so far" is accumulated.

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- The recursive nature of structures (and in particular lists) gives a way to traverse them by recursive decomposition.
- When the boundary is reached, the decomposition stops and the result is composed in a reverse of the decomposition process.
- This process can be made more efficient: introduce an extra variable in which the "result so far" is accumulated.
- When the boundary is reached this extra variable already contains the result, no need to go back and compose the final result.

Summary

- The recursive nature of structures (and in particular lists) gives a way to traverse them by recursive decomposition.
- When the boundary is reached, the decomposition stops and the result is composed in a reverse of the decomposition process.
- This process can be made more efficient: introduce an extra variable in which the "result so far" is accumulated.
- When the boundary is reached this extra variable already contains the result, no need to go back and compose the final result.
- This variable is called an accumulator.

Exercises

- Define predicates in Prolog (with accumulators) for:
  1. The length of a list
  2. The sum of elements of a list
  3. The reverse of a list
  4. The list of elements on even positions

Example: List Length

- Without accumulator:

```
% length of a list
 % boundary condition
 listlen([], 0).
 % recursion
 listlen([H|T], N):-
        listlen(T, N1),
       N is N1+1.
```

- With accumulator:

```
% length of a list with accumulators
 % call of the accumulator:
   listlen1(L, N):-
         lenacc(L, 0, N).
 % boundary condition for accumulator
   lenacc([], A, A).
 % recursion for the accumulator
   lenacc([H|T], A, N):-
          A1 is A + 1,
          lenacc(T, A1, N).
```

- With accumulator:

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% length of a list with accumulators
 % call of the accumulator:
   listlen1(L, N):-
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 % boundary condition for accumulator
   lenacc([], A, A).
 % recursion for the accumulator
   lenacc([H|T], A, N):-
          A1 is A + 1,
          lenacc(T, A1, N).
```

- Inside Prolog, for the query ?- listlen1 ([a, b, c], N):

```
lenacc([a, b, c], 0, N).
lenacc([b, c], 1, N).
lenacc([c], 2, N).
lenacc([], 3, N)
```

The return variable is shared by every goal in the trace.

Example: Reverse

- Without accumulators:

```
%% reverse
% boundary condition
  reverse1([],[]).
% recursion
  reverse1([X|TX], L):-
      reverse1(TX, NL),
      appendLists(NL, [X], L).
```

- With accumulators:

```
%% reverse with accumulators
% call the accumulator
  reverse2(L, R):-
      reverseAcc(L, [], R).
% boundary condition for the accumulator
  reverseAcc([], R, R).
% recursion for the accumulator
  reverseAcc([H|T], A, R):-
     reverseAcc(T, [H|A], R).
```

Summary

- Accumulators provide a technique to keep trace of the "result so far" (in the accumulator variable) at each step of computation, such that when the structure is traversed the accumulator contains "the final result", which is then passed to the "output variable".

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- Consider `[a, b, c | X]` - we know that this structure is a list up to a point (up to X). We call this an open list (a list with a "hole").

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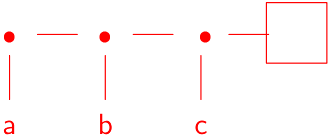
- Consider `[a, b, c | X]` - we know that this structure is a list up to a point (up to X). We call this an open list (a list with a "hole").



Using open lists

- Consider

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?- X = [a, b, c | L], L = [d, e, f, g].
   X = [a, b, c, d, e, f, g],
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- Now fill the hole with an open list:

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?- X = [a, b, c | L], L = [d, e | L1].
        X = [a, b, c, d, e|L1],
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  - the hole was filled partially.

- Now express this as a Prolog predicate:

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diff_append1(OpenList, Hole, L):-
        Hole=L.
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i.e. we have an open list (OpenList), with a hole (Hole) is filled with a list (L):

```
?- X = [a, b, c, d | Hole],
            diff_append1(X, Hole, [d, e]).
   X = [a, b, c, d, d, e],
   Hole = [d, e].
```

- Note that when we work with open lists we need to have information (i.e. a variable) both for the open list and its hole.

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- A list can be represented as the the difference between an open list and its hole.
- Notation: OpenList−Hole
  - here the difference operator − has no interpretation,
  - in fact other operators could be used instead.

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             diff_append2(X,[d, e]).
   X = [a, b, c, d, d, e]-[d, e],
   Hole = [d, e].
```

- Perhaps the fact that the answer is given as a difference list is not convenient.

- A new version that returns a(n open) list (with the hole filled) as the answer:

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diff_append3(OpenList-Hole, L, OpenList):-
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?- X = [a, b, c, d | Hole]-Hole,
             diff_append3(X,[d, e], Ans).
   X = [a, b, c, d, d, e]-[d, e],
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  - a difference list as its first argument,
  - a proper list as its second argument,
  - returns a proper list.

- A further modification – to be systematic – for this version the arguments are all difference lists:

```
diff_append4(OL1-Hole1, OL2-Hole2, OL1-Hole2):-
        Hole1 = OL2.
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and its usage:

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?- X=[a,b,c|Ho]-Ho,
diff_append4(X, [d,e,f|Hole2]-Hole2, Ans).
X = [a, b, c, d, e, f|Hole2]-[d, e, f|Hole2],
Ho = [d, e, f|Hole2],Ans = [a, b, c, d, e, f|Hol
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Ho = [d, e, f|Hole2],Ans = [a, b, c, d, e, f|Hol
```

or, if we want the result to be just the list, fill the hole with the empty list:

```
?- X=[a,b,c|Ho]-Ho,
      diff_append6(X, [d,e,f|Hole2]-Hole2,
                          Ans-[]).
   X = [a, b, c, d, e, f]-[d, e, f],
   Ho = [d, e, f],
   Hole2 = [],
   Ans = [a, b, c, d, e, f].
```

- One last modification is possible:

```
append_diff(OL1-Hole1, Hole1-Hole2, OL1-Hole2).
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its usage:

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?- X=[a,b,c|H]-H,
append_diff(X, [d,e,f|Hole2]-Hole2,
Ans-[]).
X = [a, b, c, d, e, f]-[d, e, f],
H = [d, e, f],
Hole2 = [],
Ans = [a, b, c, d, e, f].
```

Example: adding to back

- Let us consider the program for adding one element to the back of a list:

```
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- But difference lists can help - the hole is at the end of the list:

```
add_to_back_d(El,OpenList-Hole, Ans):-
append_diff(OpenList-Hole, [El|ElHole]-ElHole,
Ans-[]).
```

Problems with difference lists

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- There are also problems with the occurs check (or lack there of):

```
empty(L-L).

?- empty([a|Y]-Y).
   Y = [a|**].
```

- − in difference lists is a partial function. It is not defined for [a, b, c]−[d] :

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append_diff_fix(X-Y, Y-Z, X-Z):-
      suffix(Y, X),
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  however, now the execution time becomes linear again.