Logic Programming

Lists. Recursion

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- They are *a* data structure in Prolog.
- Lists can represent practically *any* structure.

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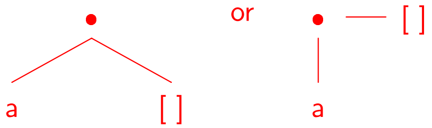
- "General case" : .(h, t) – the nonempty list, where:
  - h - the head, can be any term,
  - t - the tail, must be a list.

List representations

- .(a, [ ]) is represented as



"tree representation" *"vine representation"*

List representations (cont'd)

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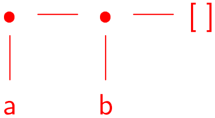

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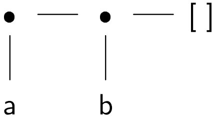
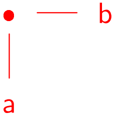
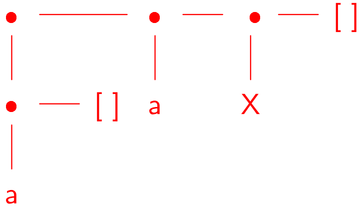


- .(a, b) is *not* a list, but it is a legal Prolog structure, represented as



List representations (cont'd)

- .(.(a, []), .(a, .(X, [ ]))) is represented as



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  [a, b],
  [[a], a, X].

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- Prolog will give:

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- Attention! [a | b] is not a list, but it is a valid Prolog expression, corresponding to .(a, b)

Unifying lists: examples

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[X, Y, Z] = [john, likes, fish]
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  [X, Y | Z] = [mary, likes, wine]
   X = mary
   Y = likes
   Z = [wine]
```

Unifying lists: examples (cont'd)

```
[[the, Y] | Z] = [[X, hare], [is, here]]
X = the
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[[the, Y] | Z] = [[X, hare], [is, here]]
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 false
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```
[white|Q] = [P|horse]
 P = white
 Q = horse
```

Strings

- In Prolog, strings are written inside double quotation marks.
- Example: "a string".
- Internally, a string is a list of the corresponding ASCII codes for the characters in the string.
- ```
  ?- X = "a string".
  X = [97, 32, 115, 116, 114, 105, 110, 103].
  ```
  ```

Summary

- Items of interest:
  - the anatomy of a list in Prolog .(h, t)
  - graphic representations of lists: "tree representation", "vine representation",
  - syntactic sugar for lists [...] ,
  - list manipulation: head-tail notation [H|T],
  - strings as lists,
  - unifying lists.

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  (b) the general case, which describes the recursive call.

Example: lists as an inductive domain

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- any other list is made of a head and a tail (the tail should be a list): [H|T].

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- For [ ] the predicate is false, therefore it will be omitted.
- Note that the recursive call is on a smaller list (second argument). The elements in the recursive call are getting smaller in such a way that eventually the computation will succeed, or reach the empty list and fail. predicate for the empty list (where it fails).

When to use the recursion?

- Avoid circular definitions:

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parent(X, Y):- child(Y, X).
child(X, Y):- parent(Y, X).
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child(X, Y):- parent(Y, X).
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- Careful with left recursion:

```
person(X):-person(Y), mother(X, Y).
person(adam).
```

In this case,

```
?-person(X).
```

will loop (no chance to backtrack). Prolog tries to satisfy the rule and this leads to the loop.

- Order of facts, rules in the database:

```
is_list([A|B]):-  is_list(B).
is_list([]).
```

  The following query will loop:

```
?- is_list(X)
```

- The order in which the rules and facts are given matters. In general, **place facts before rules**.

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  5. The concatenation of two lists.