GTC

Seminar 1

1. How many strings of the form $L_1L_2L_3D_1D_2$ exist if L_i are uppercase letters, and D_j are decimal digits?
   (Remark: assume there are 26 uppercase letters.)

2. How many strings with 7 letters exist, for each of the following constraints:

   (a) Letters can occur repeatedly?

   (b) No letter can appear more than once?

   (c) The string starts with X and letters can not occur repeatedly?

   (d) The string does not start with X and letters can appear repeatedly?

   Assume there are 26 uppercase letters.

3. A partial function from a set A to a set B is a function from a possibily empty subset of A, called the *domain* of f and denoted by $dom(f)$, to B. (We say that f is *undefined* for the elements from $A \setminus dom(f)$.) How many such partial functions exist when A has m elements and B has n elements?

4. A *palindrome* is a string that reads the same forward or reversed. How many bit strings of length n are palindromes?

5. A committee consists of members such that each of the 41 counties of Romania contributes with exactly one person, which is either a senator or a deputy from that county. We assume there are 2 senators and 3 deputies from each county. How many such committees can be made?

6. How many bit strings with length 8 start with three zeros or end with two zeros?

7. A questionnaire consists of 10 questions. Every question has four possible answers.

   (a) In how many way can we fill in such a questionnaire?

   (b) In how many way can we fill in such a questionnaire, if we are allowed not to answer some questions?

8. How many strings of five ASCII characters do contain the character `@` at least once? (REMARK: There are 128 different characters.)

9. How many strings of four decimal digits fulfil one of the following constraints:

   (a) contains every digit at least once?

   (b) starts with an odd digit?

   (c) contains digit `4` exactly two times?

10. How many permutations of the letters A,B,C,D,E,F,G,H contain

    (a) ED?

    (b) CDE?

    (c) BA and FGH?

    (d) AB, DE şi GH?

    (e) CAB and BED?

    (f) BCA and ABF?

11. Let $R_1 R_2 \cdots R_{10}$ be the outcome of flipping a coin ten times, where $R_i \in \{H, T\}$ indicates the outcome of the i-th flip: H for head, and T for tail.

    (a) How many possible outcomes are there?

    (b) How many possible outcomes contain exactly two heads?

    (c) How many possible outcomes contain exactly three tails?

    (d) How many possible outcomes contain the same number of heads and tails?

12. Find n which fulfils the following constraints:

    a) $P(n, 2) = 110$. b) $P(n, 4) = 12\,P(n, 2)$.

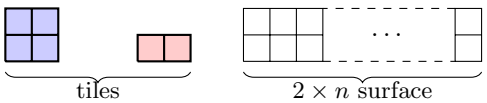
13. Find n which fulfils the following constraints:

    a) $\binom{n}{2} = 45$. b) $\binom{n}{3} = P(n, 2)$. c) $\binom{n}{5} = \binom{n}{2}$.

14. Give a combinatorial proof to $\binom{n}{r} = \binom{n}{n-r}$.

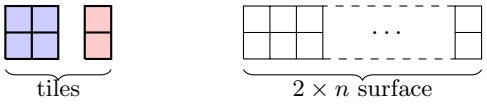
15. How many different strings can be produced by rearranging the letters of MISSISSIPPI?

16. Use the rules of sum and product to determine a recursive formula to compute the number a_n of ways in which we can cover the surface of $2 \times n$ squares illustrated below with tiles 2×2 and 1×2:



What are the values of a_1, a_2 and a_5?

17. Use the rules of sum and product to determine a recursive formula to compute the number a_n of ways in which we can cover the surface of $2 \times n$ squares illustrated below with tiles 2×2 and 2×1:



What are the values of b_1, b_2 and b_5?

18. Use the rules of sum and product to find a recursive formula to compute the number of bit strings of length n without two consecutive zeros.

19. Use the rules of sum and product to find a recursive formula to compute the number of bit strings of length n without three consecutive zeros.

20. How many strings starting with two or three letters from the set $\{A, B, C, X, Y, Z\}$ followed by two or three decimal digits do exist?