

```
In[63]= << Combinatorica`
```

---

## Generating permutations

```
In[64]= Permutations[3]
```

```
Out[64]= {{1, 2, 3}, {1, 3, 2}, {2, 1, 3}, {2, 3, 1}, {3, 1, 2}, {3, 2, 1}}
```

```
In[65]= Permutations[4]
```

```
Out[65]= {{1, 2, 3, 4}, {1, 2, 4, 3}, {1, 3, 2, 4}, {1, 3, 4, 2}, {1, 4, 2, 3}, {1, 4, 3, 2},  
          {2, 1, 3, 4}, {2, 1, 4, 3}, {2, 3, 1, 4}, {2, 3, 4, 1}, {2, 4, 1, 3}, {2, 4, 3, 1},  
          {3, 1, 2, 4}, {3, 1, 4, 2}, {3, 2, 1, 4}, {3, 2, 4, 1}, {3, 4, 1, 2}, {3, 4, 2, 1},  
          {4, 1, 2, 3}, {4, 1, 3, 2}, {4, 2, 1, 3}, {4, 2, 3, 1}, {4, 3, 1, 2}, {4, 3, 2, 1}}
```

```
In[66]= Permute[{A, B, C, D}, Permutations[3]]
```

```
Out[66]= {{A, B, C}, {A, C, B}, {B, A, C}, {B, C, A}, {C, A, B}, {C, B, A}}
```

```
In[67]= MinimumChangePermutations[{x, y, z}]
```

```
Out[67]= {{x, y, z}, {y, x, z}, {z, x, y}, {x, z, y}, {y, z, x}, {z, y, x}}
```

```
In[68]= Table[RandomPermutation[2], {10}]
```

```
Out[68]= {{1, 2}, {2, 1}, {1, 2}, {1, 2}, {2, 1}, {2, 1}, {1, 2}, {2, 1}, {2, 1}, {2, 1}}
```

```
In[69]= Permute[{a, b, c, d, e}, {5, 3, 2, 1, 4}]
```

```
Out[69]= {e, c, b, a, d}
```

```
In[70]= Permute[Permute[{a, b, c, d, e}, {1, 5, 4, 2, 3}], {4, 5, 1, 3, 2}]
```

```
Out[70]= {b, c, a, d, e}
```

```
In[71]= LexicographicPermutations[3]
```

```
Out[71]= {{1, 2, 3}, {1, 3, 2}, {2, 1, 3}, {2, 3, 1}, {3, 1, 2}, {3, 2, 1}}
```

```
In[72]= Permutations[3]
```

```
Out[72]= {{1, 2, 3}, {1, 3, 2}, {2, 1, 3}, {2, 3, 1}, {3, 1, 2}, {3, 2, 1}}
```

```
In[73]= Clear[P, Q, R, S]; LexicographicPermutations[{P, Q, R, S}]
```

```
Out[73]= {{P, Q, R, S}, {P, Q, S, R}, {P, R, Q, S}, {P, R, S, Q}, {P, S, Q, R}, {P, S, R, Q},  
          {Q, P, R, S}, {Q, P, S, R}, {Q, R, P, S}, {Q, R, S, P}, {Q, S, P, R}, {Q, S, R, P},  
          {R, P, Q, S}, {R, P, S, Q}, {R, Q, P, S}, {R, Q, S, P}, {R, S, P, Q}, {R, S, Q, P},  
          {S, P, Q, R}, {S, P, R, Q}, {S, Q, P, R}, {S, Q, R, P}, {S, R, P, Q}, {S, R, Q, P}}
```

LexicographicPermutations is slower than Permutations.

```
In[74]= Permutations[{1, 1, 2}]
```

```
Out[74]= {{1, 1, 2}, {1, 2, 1}, {2, 1, 1}}
```

```
In[75]:= LexicographicPermutations[{1, 1, 2}]
```

```
Out[75]:= {{1, 1, 2}, {1, 2, 1}, {1, 1, 2}, {1, 2, 1}, {2, 1, 1}, {2, 1, 1}}
```

Permutations deals with multisets correctly, while LexicographicPermutations does not.

## Ranking and Unranking permutations

```
In[76]:= RankPermutation[{1, 2, 3}]
```

```
Out[76]:= 0
```

```
In[77]:= RankPermutation[{5, 1, 3, 2, 4}]
```

```
Out[77]:= 98
```

```
In[78]:= RankPermutation[{5, 1, 3, 4, 2}]
```

```
Out[78]:= 99
```

```
RankPermutation[{2, 3, 1, 5, 4}]
```

```
31
```

```
In[79]:= RankPermutation[{5, 4, 3, 2, 1}]
```

```
Out[79]:= 119
```

```
In[80]:= UnrankPermutation[3, {1, 2, 3}]
```

```
Out[80]:= {2, 3, 1}
```

```
In[81]:= UnrankPermutation[3, {3, 2, 1}]
```

```
Out[81]:= {2, 1, 3}
```

```
In[82]:= RankPermutation /@ Permutations[{1, 2, 3, 4}]
```

```
Out[82]:= {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23}
```

```
In[83]:= NextPermutation[{1, 2, 3}]
```

```
Out[83]:= {1, 3, 2}
```

```
In[84]:= NextPermutation[{5, 1, 3, 2, 4}]
```

```
Out[84]:= {5, 1, 3, 4, 2}
```

```
In[85]:= NextPermutation[{5, 4, 3, 2, 1}]
```

```
Out[85]:= {1, 2, 3, 4, 5}
```

```
In[86]:= Range[5]
```

```
Out[86]:= {1, 2, 3, 4, 5}
```

```
In[87]:= Reverse[%]
```

```
Out[87]:= {5, 4, 3, 2, 1}
```

```

In[88]:= NextPermutation[%] (*NextPermutation[Reverse[Range[5]]] *)
Out[88]= {1, 2, 3, 4, 5}

In[89]:= UnrankPermutation[2, Range[3]]
Out[89]= {2, 1, 3}

In[90]:= Table[UnrankPermutation[n, Range[3]], {n, 0, 5}]
Out[90]= {{1, 2, 3}, {1, 3, 2}, {2, 1, 3}, {2, 3, 1}, {3, 1, 2}, {3, 2, 1}}

In[91]:= Table[UnrankPermutation[n, Range[3]], {n, 0, 9}]
Out[91]= {{1, 2, 3}, {1, 3, 2}, {2, 1, 3}, {2, 3, 1},
          {3, 1, 2}, {3, 2, 1}, {1, 2, 3}, {1, 3, 2}, {2, 1, 3}, {2, 3, 1}}

In[92]:= Map[RankPermutation, Permutations[{1, 2, 3, 4}]]
Out[92]= {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23}

```

---

## Generating subsets

```

In[93]:= Subsets[{1, 2, 3}]
Out[93]= {{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}

In[94]:= LexicographicSubsets[{1, 2, 3}]
Out[94]= {{}, {1}, {1, 2}, {1, 2, 3}, {1, 3}, {2}, {2, 3}, {3}}

In[95]:= Subsets[{1, 2, 3}, 2]
Out[95]= {{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}}

In[96]:= KSubsets[{1, 2, 3}, 2]
Out[96]= {{1, 2}, {1, 3}, {2, 3}}

In[97]:= KSubsets[{1, 2, 3, 4, 5}, 3]
Out[97]= {{1, 2, 3}, {1, 2, 4}, {1, 2, 5}, {1, 3, 4},
          {1, 3, 5}, {1, 4, 5}, {2, 3, 4}, {2, 3, 5}, {2, 4, 5}, {3, 4, 5}}

```

In the following we generate 13 lexicographically consecutive subsets of [a, b, c, d, e, f] starting with [a, b, d, f].

```

In[98]:= Clear[a, b, c, d, e, f];
          NestList[NextLexicographicSubset[{a, b, c, d, e, f}, #] &, {a, b, d, f}, 13]
Out[98]= {{a, b, d, f}, {a, b, e}, {a, b, e, f}, {a, b, f}, {a, c}, {a, c, d}, {a, c, d, e},
          {a, c, d, e, f}, {a, c, d, f}, {a, c, e}, {a, c, e, f}, {a, c, f}, {a, d}, {a, d, e}}

```

```
In[99]:= Clear[a, b, c, d]; BinarySubsets[{a, b, c, d}]
```

```
Out[99]:= {{}, {d}, {c}, {c, d}, {b}, {b, d}, {b, c}, {b, c, d}, {a},
           {a, d}, {a, c}, {a, c, d}, {a, b}, {a, b, d}, {a, b, c}, {a, b, c, d}}
```

```
In[100]:= NestList[NextBinarySubset[{a, b, c, d}, #] &, {}, 15]
```

```
Out[100]:= {{}, {d}, {c}, {c, d}, {b}, {b, d}, {b, c}, {b, c, d}, {a},
            {a, d}, {a, c}, {a, c, d}, {a, b}, {a, b, d}, {a, b, c}, {a, b, c, d}}
```

$n$  is taken modulo the number of subsets, therefore one can use any positive or negative number to specify the rank of a subset.

```
In[101]:= UnrankBinarySubset[-10, {a, b, c, d}]
```

```
Out[101]:= {b, c}
```

```
In[102]:= UnrankSubset[2, {a, b, c, d}]
```

```
Out[102]:= {b}
```

```
In[103]:= UnrankSubset[0, {a, b, c, d}]
```

```
Out[103]:= {}
```

```
In[104]:= Table[UnrankSubset[n, {a, b, c, d}], {n, 0, 15}]
```

```
Out[104]:= {{}, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d},
            {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c, d}}
```

```
In[105]:= Subsets[{1, 2, 3}]
```

```
Out[105]:= {{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}
```

```
In[106]:= LexicographicSubsets[{1, 2, 3}]
```

```
Out[106]:= {{}, {1}, {1, 2}, {1, 2, 3}, {1, 3}, {2}, {2, 3}, {3}}
```

```
In[107]:= RankSubset[{1, 2, 3}, {2}]
```

```
Out[107]:= 2
```

```
In[108]:= RankSubset[{1, 2, 3}, {1, 2}]
```

```
Out[108]:= 4
```

```
In[109]:= RankKSubset[{2}, {1, 2, 3}]
```

```
Out[109]:= 1
```

```
In[110]:= NextSubset[{1, 2, 3}, {2}]
```

```
Out[110]:= {3}
```

```
In[111]:= NextSubset[{1, 2, 3}, {1, 3}]
```

```
Out[111]:= {2, 3}
```

```
In[112]:= UnrankSubset[3, {1, 2, 3}]
```

```
Out[112]= {3}
```

```
In[113]:= UnrankSubset[2, {1, 2, 3}]
```

```
Out[113]= {2}
```