Revision

Permutations

Rank

Recurrence

Method for solving recurrence relations

rewriting a_n as a function of n

Steps :

1. Identify : $k, c_1, c_2, ..., c_k$

2. Write the characteristic equation $r^k - c_1 r^{k-1} - ... - c_k = 0$.

3. Compute the roots of the characteristic equation r_1, r_2, r_3

4. Assign the multiplicity for each of the roots $m_1 + m_2 + ... = k$.

E.g., if we have one single root r_1 and $k = 2$, then $m_1 = 2$,
e.g., if we have 2 roots r_1, r_2 and $k = 3$, $m_1 = 1$, $m_2 = 2$

e.g., r_1, r_2, r_3, $k = 5$, $m_1 + m_2 + m_3 = 5$, $m_1 = 2$, $m_2 = 2$, $m_3 = 1$

5. Write the general form of a_n using the coefficients $\alpha_{1,0}$, ...

6. Find the values of $\alpha_{1,0}$, $\alpha_{1,1}$, $\alpha_{2,0}$, ... using the initial conditions a_0, a_1, a_2 ...

Ex.7

$a_n = 4 a_{n-1} - 4 a_{n-2}$, for $n \geq 0$

$a_0 = 6$

$a_1 = 8$

1. $k = 2$, $c_1 = 4$, $c_2 = c_k = -4$

2. $r^2 - 4 r^1 + 4 = 0$

3. $r_1 = 2$, $(r - 2)^2$

$\Delta = b^2 - 4 ac = 16 - 16 = 0$

$r_{1,2} = (-b + - sqr[\Delta]) / 2a$

$r_1 = 4/2 = 2$

4. $m_1 = 2$

5. $a_n = (\alpha_{1,0} + \alpha_{1,1}\, n)\, 2^n$

6. $a_0 = (\alpha_{1,0} + \alpha_{1,1}\, 0)\, 2^0 = 6$
   $a_0 = \alpha_{1,0} = 6$
   $a_1 = (\alpha_{1,0} + \alpha_{1,1})\, 2 = 8/2$

$6 + \alpha_{1,1} = 4, \ \alpha_{1,1} = -2$

$a_n = (6 - 2n)\, 2^n$

Polya

a. 1 perm : (1) (2) (3) (4) (5) (6) $\longrightarrow x_1^6$

b. 2 perm : (1, 2, 3, 4, 5, 6), (1, 6, 5, 4, 3, 2) $\longrightarrow 2\, x_6$

c. 2 perm : (1, 3, 5) (2, 4, 6), (1, 5, 3) (2, 6, 4) $\longrightarrow 2\, x_3^2$

d. 4 perm : (1, 4) (2, 5) (3, 6), (1, 6) (2, 5) (3, 4),
(1, 2) (3, 6) (4, 5), (1, 4) (2, 3) (5, 6) $\longrightarrow 4\, x_2^3$

e. 3 perm : (1) (2, 6) (3, 5) (4), (1, 3) (2) (4, 6) (5), (1, 5) (2, 4) (3) (6) $\longrightarrow 3\, x_1^2\, x_2^2$

12 permutations in total

I. Compute the cycle index

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In[1]:= PG[x1_, x2_, x3_, x4_, x5_, x6_] := (x1^6 + 2 x6 + 2 x3^2 + 4 x2^3 + 3 x1^2 x2^2) / 12
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II. Compute the pattern inventory for coloring with 2 colours (y1, y2)

Replace in P_G : $x1 \longrightarrow y1 + y2, \ x2 \longrightarrow y1^2 + y2^2, \ \ldots.$

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In[2]:= FG = PG[y1 + y2, y1^2 + y2^2, y1^3 + y2^3, y1^4 + y2^4, y1^5 + y2^5, y1^6 + y2^6]
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Out[2]= $\frac{1}{12}\left((y1 + y2)^6 + 3\,(y1 + y2)^2\,(y1^2 + y2^2)^2 + 4\,(y1^2 + y2^2)^3 + 2\,(y1^3 + y2^3)^2 + 2\,(y1^6 + y2^6)\right)$

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In[3]:= Expand[FG]
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Out[3]= $y1^6 + y1^5\, y2 + 3\, y1^4\, y2^2 + 3\, y1^3\, y2^3 + 3\, y1^2\, y2^4 + y1\, y2^5 + y2^6$

For coloring D_6 using exactly 2 times color y1 and 4 times the color y2 we are looking in F_G for the coefficient of : $y1^2\, y2^4$

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In[4]:= Coefficient[FG, y1^2 y2^4]
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Out[4]= 3