	Start	1	2	3	4	5	6	7	8	9	10
GRADE:											

Graph Theory and Combinatorics
Partial Exam (Combinatorics)

Name:

1. (1.25p) Rank and Canonical cyclic structure
   (a) (0.5p) Given the permutation $\langle 1, 6, 2, 3, 5, 4\rangle$. Compute step by step (and describe each step for obtaining) the next permutation in lexicographic order.
   (b) (0.5p) Write down the permutation with the following cyclic structure $(4, 7, 3)(8, 5, 1, 2)(10, 6, 9)$
   (c) (0.25p) Give the canonical cyclic structure of the cyclic structure $(4, 7, 3, 5)(2, 1, 6, 8)(9, 10)$.
2. (1p) How many ways are there to select 7 bills from a cash box containing bills of \$1, \$2, \$5, \$10, \$20, \$50. Assume that: the order in which the bills are chosen does not matter; the bills are indistinguishable; there are at least 7 bills of each type.
3. (0.75p) How many different strings can be produced by rearranging the letters of VERSAILLES?
4. (0.5p) Compute the 6-permutation with repetition of the set $\{1, 2, 3, 4\}$ with rank 28 in the lexicographic ordering.
5. (0.5p) Compute the rank of the permutation $\langle 5, 1, 3, 2, 4\rangle$ in lexicographic order.
6. (1p) Types of permutations
   (a) (0.25p) Which of the following types is a valid permutation type?
      i. $[0, 1, 0, 1, 1]$
      ii. $[3, 1, 1, 0, 0, 0]$
      iii. $[0, 0, 0, 1]$
   (b) (0.5p) Write all the plausible types of the permutations of the set $\{1, 2, 3, 4\}$.
   (c) (0.25p) How many permutations have the same type as the permutation $\langle 5, 2, 4, 1, 3, 6\rangle$.

7. (1p) Solve the following recurrence relation together with the initial conditions given:
$a_n = 4\,a_{n-1} - 4\,a_{n-2}$, for $n \geq 0$, $a_0 = 6$, $a_1 = 8$.

8. (0.5p) Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive zeros. How many such bit strings of length 5 do we have?

9. (1.5p) What is the general form of a solution of the linear nonhomogeneous recursive relation $a_n = 6\,a_{n-1} - 12\,a_{n-2} + 8\,a_{n-3} + F(n)$, if $F(n) = n\,2^n$?

10. (1p) Given the dihedral group D_6 which consists of the following symmetries:

    $(1)(2)(3)(4)(5)(6), (1,2,3,4,5,6), (1,3,5)(2,4,6), (1,4)(2,5)(3,6),$
    $(1,5,3)(2,6,4), (1,6,5,4,3,2), (1,6)(2,5)(3,4), (1,2)(3,6)(4,5),$
    $(1,4)(2,3)(5,6), (1)(2,6)(3,5)(4), (1,3)(2)(4,6)(5), (1,5)(2,4)(3)(6)$

    (a) (0.25p) Compute the cycle index of D_6;
    (b) (0.5p) Compute the pattern inventory polynomial for coloring with 2 colors;
    (c) (0.25p) How many possibilities there are to color D_6 by using exactly two times the first color and four times the second color?

Start: 1p