

Automated Theorem Proving, SS 2022. Seminar 5

1. Construct the sequent proof for the following formulae

(a) $((A \vee B) \Rightarrow C) \Rightarrow ((A \Rightarrow C) \wedge (B \Rightarrow C)),$

(b) $((A \wedge B) \Rightarrow C) \Rightarrow ((A \Rightarrow C) \vee (B \Rightarrow C)),$

(c) $((A \Rightarrow C) \wedge (B \Rightarrow C)) \Leftrightarrow ((A \vee B) \Rightarrow C)$

(d) $((A \Rightarrow B) \wedge (B \Rightarrow C)) \Rightarrow ((A \wedge B) \Rightarrow C)$

(e) $((B \Rightarrow A) \wedge (B \Rightarrow C)) \Rightarrow ((A \vee B) \Rightarrow C)$

in two ways:

1.1 by using the special rules (resolution, subsumption, implication) and unique goal,

1.2 by unit propagation

2. For the following expression (which is similar to the way we do Mathematics in practice), identify which symbols represent: constants, variables, functions, predicates, and which parts are terms or formulae. Express it in standard predicate logic notation according to the definition of syntax, by introducing new predicate symbols and new function symbols where necessary.

$$\forall_{f,a} C[f,a] \Leftrightarrow \forall_{\epsilon>0} \exists_{\delta>0} \forall_x (|x-a| < \delta \Rightarrow |f[x] - f[a]| < \epsilon)$$

3. Construct the representing tree of the following formula and compute the depth of it:

$$\forall_x ((\neg P[x, f[x]] \vee R[x, f[x], g[x]]) \wedge (Q[x, g[x]] \vee R[x, f[x], g[x]]))$$