Automated Theorem Proving, SS 2022. Seminar 2

1. For each of the following formulas determine whether is valid/invalid/satisfiable/unsatisfiable or some combination of these by using equivalent transformations.

   (a) $(P \Rightarrow Q) \Rightarrow (\neg Q \Rightarrow \neg P)$

   (b) $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$

   (c) $P \vee (P \Rightarrow Q)$

   (d) $(P \wedge (Q \Rightarrow P)) \Rightarrow P$

   (e) $P \vee (Q \Rightarrow \neg P)$

   (f) $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow ((P \wedge Q) \Rightarrow R)$

   (g) $((Q \Rightarrow P) \wedge (Q \Rightarrow R)) \Rightarrow ((P \vee Q) \Rightarrow R)$

   (h) $(P \vee \neg Q) \wedge (\neg P \vee Q)$

   (i) $\neg P \wedge (\neg (P \Rightarrow Q))$

   (j) $P \Rightarrow \neg P$

   (k) $\neg P \Rightarrow P$

2. Write the tables of the boolean functions corresponding to $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$. Using them, determine the truth value of:

   - The formula $(A \wedge (A \Rightarrow B)) \Rightarrow B$ under the interpretation $I = \{A \to \mathbb{T}, B \to \mathbb{F}\}$.
   - The formula $(P \Rightarrow Q) \Longleftrightarrow (\overline{Q} \Rightarrow \overline{P})$ under the interpretation $I = \{P \to \mathbb{F}, Q \to \mathbb{F}\}$.
   - The formula $((A \vee B) \Rightarrow C) \Longleftrightarrow ((A \Rightarrow C) \wedge (B \Rightarrow C))$ under the interpretation $I = \{A \to \mathbb{T}, B \to \mathbb{T}, C \to \mathbb{F}\}$.

   (*Hint: The tables of boolean functions corresponding to* $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ *correspond to the tables which we have outlined in the first lab and used for defining the semantics of logical connectives.*)

3. Is it possible to have a formula that is both in conjunctive and disjunctive normal form. If so, give 5 examples.

4. Prove by reduction to CNF the semantic equivalence between $(A \wedge B) \Rightarrow C$ and $(A \Rightarrow C) \vee (B \Rightarrow C)$.

5. Transform the following into disjunctive normal form

   (a) $(P \Rightarrow Q) \Rightarrow R$

   (b) $\neg (P \wedge Q) \wedge (P \vee Q)$

6. Transform the following into conjunctive normal form

   (a) $(P \Rightarrow Q) \Rightarrow R$

   (b) $(\neg P \wedge Q) \vee (P \wedge \neg Q)$