Automated Theorem Proving, SS 2021. Seminar 1

1. Give an example of a mathematical result which had a important impact on real life.

2. Give an example of a software failure which had an important negative impact in real life.

3. For each of the following formulas determine whether is valid/invalid/satisfiable/unsatisfiable or some combination of these. Use the truth table method and then use equivalent transformations.

   (a) $(P \Rightarrow Q) \Rightarrow (\neg Q \Rightarrow \neg P)$

   (b) $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$

   (c) $P \vee (P \Rightarrow Q)$

   (d) $(P \wedge (Q \Rightarrow P)) \Rightarrow P$

   (e) $P \vee (Q \Rightarrow \neg P)$

   (f) $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow ((P \wedge Q) \Rightarrow R)$

   (g) $((Q \Rightarrow P) \wedge (Q \Rightarrow R)) \Rightarrow ((P \vee Q) \Rightarrow R)$

   (h) $(P \vee \neg Q) \wedge (\neg P \vee Q)$

   (i) $\neg P \wedge (\neg (P \Rightarrow Q))$

   (j) $P \Rightarrow \neg P$

   (k) $\neg P \Rightarrow P$

4. Define the meta-function $\text{Vars}[\varphi]$ which gives set of propositional variables of the propositional formula φ. (Hint: use the induction principle suggested by the definition of propositional logic formulas.) Examples: $\text{Vars}[\mathbb{F}] = \emptyset$, $\text{Vars}[A] = \{A\}$, $\text{Vars}[P \Rightarrow \mathbb{T}] = \{P\}$, $\text{Vars}[(P \Rightarrow Q) \Rightarrow (P \wedge Q)] = \{P, Q\}$, $\text{Vars}[Q \Rightarrow Q] = \{Q\}$

5. Using the induction principle from the syntactic definition of propositional formulae, define the meta-function $L[\varphi]$ which gives the length of the propositional formula φ.

6. Using the induction principle from the syntactic definition of propositional formulae, define the meta-function $D[\varphi]$ which gives the depth of the propositional formula φ (that is the depth of the tree which represents the formula).

7. Using the induction principle from the syntactic definition of propositional formulae and the definitions above, prove that $D[\varphi] < L[\varphi]$ for any propositional formula φ. (Where $L[\varphi]$ gives the length of the propositional formula φ.)