Automated Theorem Proving, SS 2020. Seminar 7

1. Find the truth value of the formula $F : \underset{x}{\forall}(P[x] \Rightarrow Q[f[x], a])$ under the interpretation:

$$I : \begin{cases} D = \{1,2\} \\ a_I = 1 \\ f_I : D \to D & \begin{cases} f_I[1] = 1 \\ f_I[2] = 1 \end{cases} \\ P_I : D \to \{\mathbb{T}, \mathbb{F}\} & \begin{cases} P_I[1] = \mathbb{T} \\ P_I[2] = \mathbb{F} \end{cases} \\ Q_I : D^2 \to \{\mathbb{T}, \mathbb{F}\} & \begin{cases} Q_I[1,1] = \mathbb{T} \quad Q_I[1,2] = \mathbb{F} \\ Q_I[2,1] = \mathbb{F} \quad Q_I[2,2] = \mathbb{T} \end{cases} \end{cases}$$

(Hint: In the lecture notes you have Example 3 at page 8 partially solved. Consider the case when $x \to 2$.)

2. Bring the following formulae into Skolem Standard Form:

(a)

$$\underset{x}{\forall}\underset{y}{\exists}\underset{z}{\exists}\Big(\big(\neg P[x,y] \wedge Q[x,z]\big) \vee R[x,y,z]\Big)$$

(b)

$$\underset{x}{\forall}\underset{y}{\forall}\Big(\big(\underset{z}{\exists}P[x,z] \wedge P[y,z]\big) \Rightarrow \underset{u}{\exists}Q[x,y,u]\Big).$$

(Hint: see the examples from the lecture notes.)

3. Transform the formulae φ_1, φ_2, φ_3, φ_4, and $\neg\psi$ into a set of clauses, where:

$$\varphi_1 : \quad \underset{x}{\forall}\underset{y}{\forall}\underset{z}{\exists}P[x,y,z]$$

$$\varphi_2 : \quad \underset{x}{\forall}\underset{y}{\forall}\underset{z}{\forall}\underset{u}{\forall}\underset{v}{\forall}\underset{w}{\forall}\Big(((P[x,y,u] \wedge P[y,z,v] \wedge P[u,z,w]) \Rightarrow P[x,v,w]) \wedge$$
$$((P[x,y,u] \wedge P[y,z,v] \wedge P[x,v,w]) \Rightarrow P[u,z,w])\Big)$$

$$\varphi_3 : \quad \underset{x}{\forall}(P[x,e,x] \wedge P[e,x,x])$$

$$\varphi_4 : \quad \underset{x}{\forall}(P[x,i[x],e] \wedge P[i[x],x,e])$$

$$\psi : \quad \Big(\underset{x}{\forall}P[x,x,e]\Big) \Rightarrow \underset{u}{\forall}\underset{v}{\forall}\underset{w}{\forall}(P[u,v,w] \Rightarrow P[v,u,w])$$

4. Prove by resolution that ψ is a logical consequence of φ_1 and φ_2 where:

$$\varphi_1 : \quad \underset{x}{\exists}\Big(P[x] \ \wedge \ \underset{y}{\forall}(D[y] \Rightarrow L[x,y])\Big)$$

$$\varphi_2 : \quad \underset{x}{\forall}\Big(P[x] \Rightarrow \underset{y}{\forall}(Q[y] \Rightarrow \neg L[x,y])\Big)$$

$$\psi : \underset{x}{\forall}(D[x] \Rightarrow \neg Q[x])$$

(Hint: First transform the formulae $\varphi_1, \varphi_2, \neg\psi$ into their clausal form, and then apply resolution to the obtained set of clauses.)

5. Prove by resolution that ψ is a logical consequence of φ_1, φ_2, and φ_3 where:

$$\varphi_1 : \quad \underset{x}{\forall}(Q[x] \Rightarrow \neg P[x])$$

$$\varphi_2 : \quad \underset{x}{\forall}\Big((R[x] \wedge \neg Q[x]) \Rightarrow \underset{y}{\exists}(T[x,y] \wedge S[y])\Big)$$

$$\varphi_3 : \underset{x}{\exists}\Big(P[x] \wedge \underset{y}{\forall}(T[x,y] \Rightarrow P[y]) \wedge R[x]\Big)$$

$$\psi : \underset{x}{\exists}(S[x] \wedge P[x])$$

(Hint: First transform the formulae $\varphi_1, \varphi_2, \varphi_3, \neg\psi$ into their clausal form, and then apply resolution to the obtained set of clauses.)