Automated Theorem Proving, SS 2020. Homework 3

1. Write the tables of the boolean functions corresponding to $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$. Using them, determine the truth value of:

   - The formula $(A \wedge (A \Rightarrow B)) \Rightarrow B$ under the interpretation $I = \{A \to \mathbb{T}, B \to \mathbb{F}\}$.
   - The formula $(P \Rightarrow Q) \iff \left(\overline{Q} \Rightarrow \overline{P}\right)$ under the interpretation $I = \{P \to \mathbb{F}, Q \to \mathbb{F}\}$.
   - The formula $((A \vee B) \Rightarrow C) \iff ((A \Rightarrow C) \wedge (B \Rightarrow C))$ under the interpretation $I = \{A \to \mathbb{T}, B \to \mathbb{T}, C \to \mathbb{F}\}$.

   (*Hint: The tables of boolean functions corresponding to $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ correspond to the tables which we have outlined in the first lab and used for defining the semantics of logical connectives.*)

2. Is it possible to have a formula that is both in conjunctive and disjunctive normal form. If so, give 5 examples.

3. Prove by reduction to CNF the semantic equivalence between $(A \wedge B) \Rightarrow C$ and $(A \Rightarrow C) \vee (B \Rightarrow C)$.

4. Apply the resolution principle to the following set of clauses:
   $A \vee B,\ \overline{A} \vee C \vee D,\ C \vee \overline{D},\ \overline{B} \vee \overline{C},\ \overline{A} \vee B \vee \overline{C},\ A \vee \overline{B} \vee C \vee D.$

5. Apply the DPLL algorithm to the set of clauses from the previous exercise.

6. Using the induction principle from the syntactic definition of propositional formulae, define the meta-function $D[\varphi]$ which gives the depth of the propositional formula φ (that is the depth of the tree which represents the formula).

7. Using the induction principle from the syntactic definition of propositional formulae and the definitions above, prove that $D[\varphi] < L[\varphi]$ for any propositional formula φ. (Where $L[\varphi]$ gives the length of the propositional formula φ.)

8. Prove that for any propositional formulae $\varphi_1, \varphi_2, \ldots, \varphi_n, \psi$, if $\varphi_1, \varphi_2, \ldots, \varphi_n \models \psi$, then $(\varphi_1 \wedge \varphi_2 \wedge \ldots \wedge \varphi_n) \Rightarrow \psi$ is valid. (See the style used in the lecture for proving the opposite implication.)