

Automated Theorem Proving, SS 2019. Homework 2 (due 27 March 2019)

1. Write the tables of the boolean functions corresponding to \neg , \wedge , \vee , \Rightarrow , \Leftrightarrow . Using them, determine the truth value of:

- The formula $(A \wedge (A \Rightarrow B)) \Rightarrow B$ under the interpretation $I = \{A \rightarrow \mathbb{T}, B \rightarrow \mathbb{F}\}$.
- The formula $(P \Rightarrow Q) \Leftrightarrow (\neg Q \Rightarrow \neg P)$ under the interpretation $I = \{P \rightarrow \mathbb{F}, Q \rightarrow \mathbb{F}\}$.
- The formula $((A \vee B) \Rightarrow C) \Leftrightarrow ((A \Rightarrow C) \wedge (B \Rightarrow C))$ under the interpretation $I = \{A \rightarrow \mathbb{T}, B \rightarrow \mathbb{T}, C \rightarrow \mathbb{F}\}$.

(Hint: The tables of boolean functions corresponding to \neg , \wedge , \vee , \Rightarrow , \Leftrightarrow correspond to the tables which we have outlined in the first lab and used for defining the semantics of logical connectives.)

2. Is it possible to have a formula that is both in conjunctive and disjunctive normal form. If so, give 5 examples.
3. Construct the sequent proof (proof tree) for $(A \vee B) \Rightarrow C \models (A \Rightarrow C) \wedge (B \Rightarrow C)$.
4. In the “ $\neg\wedge$ -calculus”, prove the correctness of the following inference rules:

$$(a) \quad \frac{\Phi, \varphi_1, \varphi_2 \vdash \Psi}{\Phi, \varphi_1 \wedge \varphi_2 \vdash \Psi} \quad (\wedge \vdash)$$

$$(b) \quad \frac{\Phi, \varphi \vdash \Psi}{\Phi \vdash \neg\varphi, \Psi} \quad (\vdash \neg)$$

5. Using the “ $\neg\wedge$ -calculus”, derive the sequent rules for

- (a) disjunction in the assumption
- (b) disjunction in the goal
- (c) implication in the assumption
- (d) implication in the goal