The State of the Art of Nurse Rostering

Edmund Burke[1], Patrick De Causmaecker[2], Greet Vanden Berghe[2],
Hendrik Van Landeghem[3]

[1]School of Computer Science & IT, University of Nottingham, Jubilee Campus
Nottingham NG8 1BB, UK, Tel: +44 115 9514206
Fax: +44 115 9514254, e-mail: ekb@cs.nott.ac.uk

[2]KaHo St.-Lieven, Information Technology, Gebr. Desmetstraat 1
9000 Gent, Belgium, Tel: +32 9 2658610, Fax: +32 9 2256269
e-mail: {patrick.decausmaecker,greet.vandenberghe}@kahosl.be

[3]Vakgroep Technische Bedrijfsvoering, Universiteit Gent, Technologiepark 903
9052 Gent-Zwijnaarde, Belgium, Tel: +32 9 2645502
Fax: +32 9 2645847, e-mail: hendrik.vanlandeghem@ugent.be

Abstract

Nurse rostering is a complex scheduling problem that affects hospital personnel on a daily basis all over the world. The need for quality software solutions is acute for a number of reasons. In particular, it is very important to efficiently utilise time and effort, to evenly balance the workload among people and to attempt to satisfy personnel preferences. A high quality roster can lead to a more contented and thus more effective workforce.

In this review, we discuss nurse rostering within the global personnel scheduling problem in healthcare. After considering some global approaches and previously written overviews, we present comparisons for measures that determine the complexity of nurse rostering problems. We describe and discuss solution approaches which span the interdisciplinary spectrum from operations research techniques to artificial intelligence methods. We conclude by drawing on the strengths and weaknesses of the literature to outline the key issues that need addressing in future nurse rostering research.

Keywords: nurse rostering, hospital personnel scheduling

1 Introduction

Employee scheduling has been addressed by personnel managers, operations researchers and computer scientists for more than 40 years. The domain often covers staffing, budgeting and short-term scheduling problems. Although these fields each have different time horizons, they are strongly interrelated. The scheduling of hospital personnel is particularly challenging because of different staffing needs on different days and shifts. Unlike many other organisations, healthcare institutions work around the clock. Irregular shift work has its implications on the nurses' well being and job satisfaction [92, 100]. The extent to which the staff roster satisfies the staff can affect the working environment [7, 72, 89, 90, 137].

Until recently, most personnel scheduling problems in hospitals were solved manually. This is sometimes called 'self-scheduling' in the literature (e.g. [87, 111, 118]). Scheduling by hand used to be a very time consuming task. Planners had no automatic tool to test the quality of a constructed schedule. They made use of very straightforward constraints on working time and idle time in the recurring process. It is even the case that many of the hospitals with computerised scheduling systems are not exploiting the full range of capability that is possible. Scheduling editing features are often used but automatic schedule generation features are still not common.

The importance of a systematic approach to create good timetables is very high, especially in healthcare, where it is unacceptable not to fully support patient care needs and staff requirements. Automated approaches hold significant potential for improving the timetabling process and the quality of those timetables. Mathematical or heuristic approaches can easily produce a number of solutions, they can report on the quality of schedules, they can try to divide the work evenly among personnel. One of the largest benefits of automating the personnel scheduling process is a very considerable time-saving for the administrative staff involved.

Although the main overall goal of hospital personnel scheduling and other employee scheduling problems is the same, the problems often differ considerably and so do the approaches to solve them. In this article, we present an overview of nurse rostering both from the problem formulation (Section 2) and from the solution method perspective (Section 3). In this paper, we concentrate on Nurse Rostering. However, we have to point out that the term 'rostering' has several different meanings across the literature. Wren [140] defined rostering as "the placing, subject to constraints, of resources into slots in a pattern. One may seek to minimise some objective, or simply to obtain a feasible allocation. Often the resources will rotate through a roster". In the literature, 'nurse rostering' often refers to the short term timetabling of staff (with a typical time horizon of a few weeks). In this article, we will use the term nurse rostering to refer to this process (the allocation of nurses to periods of work over several weeks).

In general hospital personnel scheduling, different approaches exist for various time horizons. Since this article concentrates on the short-term timetabling part of the problem, we will only briefly discuss longer term management decision making (which we refer to as staffing). Section 2.2 especially stresses the differences between nurse rostering and longer term planning.

In this paper we will employ the following terminology:

- **Planning Period.** The planning period is the time interval over which the staff have to be scheduled. A typical length of the planning period is 4 weeks.
- **Skill Category.** This determines a class of staff who have a particular level of qualification, skill or responsibility.
- **Shift Type.** Shift types are hospital duties which usually have a well-defined start and end time. Many nurse rostering problems are concerned with the three traditional early (e.g. 7:00-15:00), late (15:00-22:00), and night (22:00-7:00) shifts (or slight variants thereof).
- **Coverage Constraints.** Coverage constraints express the number of personnel needed for every skill category and for every shift or time interval during the entire planning period. They are often referred to as personnel demands or personnel requirements. It is very common in hospitals that the coverage constraints have been determined by a sort of workload measurement system (e.g. patient classification system), nurse-patient ratios or by a forecasting method.
- **Time Related Constraints.** These constraints refer to all the restrictions on personal schedules. All the personal requests, personal preferences, and constraints on balancing the workload among personnel belong to this category.
- **Hard Constraints** are those that must be satisfied at all costs.
- **Soft Constraints** are those that are desirable but which may need to be violated in order to generate a workable solution.
- **Work Regulations.** The contract that personnel members have with the hospital is called a 'work regulation'. It sets a number of time related constraints for the nurses.

As an illustrative example of a nurse rostering problem to highlight how this terminology is used, we consider a hospital ward with the following members of staff (and their particular *skill category*): 1 head nurse, 15 regular nurses, 3 caretakers and 2 trainees. *Shift types* include early, day, late and night shifts. Among the personnel, some people's *work regulations* are full time (which means that the person should work for example 38 hours a week, maximum 6 assignments to the night *shift type*, maximum 2 weekends per month); other *work regulations* are half time (where there are is a maximum of 10 assignments per month and at most 20 hours per week). Some nurses even work to a personal *work regulation* (for example restricting the number of Wednesday afternoon shifts). Apart from the *time related constraints* that are defined by the *work regulations*, every single nurse can specify preferences (such as a request for a particular day off). The *work regulation* of the trainees indicates, for example, that a trainee should only be assigned to a *shift type* when the supervisor is assigned to the same *shift type*. *Coverage constraints* are generally imposed upon the ward by the management. Every 4 weeks the head nurse needs to create a roster. In order to generate a feasible solution, the *coverage constraints* for every *skill category* and for every single day (and *shift type*) of the 4 week *planning period* need to be met (we refer to the *coverage constraints* as *hard constraints* in this example). While doing that, the head nurse will attempt to minimise the number of violations of *time related constraints* (which we call *soft constraints* in this example). There is an extremely high number of possible solutions to this rather simple example of a nurse rostering problem.

The motivation for this article is twofold. Firstly, we aim to provide a comprehensive overview of the current scientific state-of-the-art in automated nurse rostering and to critically review the extensive collection of scientific literature that has been generated over the last forty years. We aim to highlight the key scientific achievements in that time. Secondly, by analysing the current state of the art, we will outline what we believe to be the important research issues that have not been fully addressed by the scientific community. We will present some of the problems and issues that should be addressed in the next few years.

2 Nurse Scheduling - Description of Models

The terms nurse rostering or nurse scheduling have been used over the years to cover several types of personnel scheduling problem. In the literature (e.g. [22, 117, 127, 131]), the terms 'staffing' and 'rostering' are used to handle different decision levels. The adoption of manual or automated scheduling methods or whether to employ cyclical or non-cyclical scheduling are important decisions which lead to completely different solutions. In this section, we will concentrate on such strategic decisions. We will begin by briefly discussing previous survey articles. Most of them specifically tackle personnel scheduling in healthcare.

2.1 Literature Overviews

In an influential early overview, **Warner** [131], (1976), distinguishes 3 major areas of manpower decision research: staffing, scheduling and reallocation of nurses. Five different criteria are defined for the scheduling part of the problem:

- coverage: how different the required and the scheduled number of people for a task are.
- quality: how fair schedules are, what the work stretch length is.
- stability: how the nurses perceive the schedules (consistent, predictable on/off days and weekend work).
- flexibility: how well the system can adapt to changes in the environment.
- cost: how many resources are consumed in making the decision: e.g. personnel manager's time, computer time.

It is very interesting to combine these criteria for evaluating schedules since they address more than computable standards. From a general hospital scheduling point of view, it makes sense to take such a broad interpretation of cost (to generate the schedule) into account. However, it would also make sense to add other criteria (like 'personnel cost', for example) to the list. Nearly all the criteria are very hard to measure. Warner compares three scheduling approaches against these 5 criteria:

- In the Traditional Approach, the schedules are generated by hand. This policy is flexible, it is the only advantage with respect to criteria.
- Cyclical Scheduling generally provides good schedules but it cannot easily address personal requests.
- Computer Aided Traditional Scheduling enables a fast and more complete search for good schedules. The advantages of this approach are high with respect to all the criteria considered.

Warner's overview is oriented towards techniques for determining desired/required staffing levels, which are also briefly discussed in Section 2.2.

Fries [56], (1976), presents a bibliography of early methods for personnel rostering in healthcare institutions. Many of these early approaches rely on manual procedures, following a set of arbitrary rules. They are too restricted to be directly applicable for large modern hospitals with today's complexity. However, there is always the possibility of hybridising early approaches (or some features of early approaches) with more sophisticated modern techniques to produce even better methods.

Tien and Kamiyama [127], (1982), present a list of personnel scheduling algorithms, which are not restricted to healthcare. Many of them are based on arbitrary trial and error methods. Tien and Kamiyama concentrate on the hospital scheduling situation in the United States. A particularly interesting contribution is that they decompose the 'manpower' scheduling problem into five separated stages: determination of temporal manpower requirements, total manpower requirement, recreation blocks, recreation/work schedule, and shift schedule. Stage 1 and stage 2 are management decisions (also called the 'manpower allocation problem'), which belong to the long-term staffing part of the problem. Both stages consider defining hospital requirements and selecting resources. In the classification of [127], stages 3 to 5 include the entire short-term timetabling part of the problem, that takes preferences and constraints on personal schedules into account. Tien and Kamiyama were able to classify many papers in their 5 stage model, some covering a number of stages simultaneously. However, we believe that this division is often too simplified to capture all the problem specific features of modern nurse rostering problems.

Sitompul and Randhawa [119], (1990) concentrate on financial cost. The goal is to reduce the personnel cost. Characteristics of manpower scheduling in hospitals are fluctuating demand, human effort (which cannot be inventoried), and critical customer convenience, while the schedules are subject to different kinds of constraints. Sitompul and Randhawa define four stages in nurse scheduling:

- Determine a set of feasible schedules that satisfy the constraints.
- Select the best schedule in terms of cost, coverage, and/or other criteria.
- Fine tune to accommodate changes.
- Make specific shift assignments.

It is interesting to compare schedules with respect to the perceived quality by the personnel members instead of using the violation of constraints as a criterion. In practice, it often makes no sense to separate specific shift assignments from the schedule design because assignment to different people really influences the quality of the schedule. Sitompul and Randhawa advocate the approach of tackling staffing and rostering at the same time. They argue that separating the rostering from management decisions leads to sub-optimal schedules. From a theoretical point of view, this is absolutely true. However, we believe that a general scheduling procedure would not work without significant changes in working practices for the following reasons:

- Even though there is a high fluctuation in patient needs, it is not recommended to shift personnel around the hospital each time the request does not match the available staff. This would be the consequence if the problem were looked at from a purely global point of view.
- People prefer to express personal preferences with respect to work and free time. These preferences differ from month to month. Planners only seem to grant personnel wishes if they know the people in person.
- The problems are nearly all over-constrained and too complex to find an optimal solution in a reasonable amount of time. Splitting them up cannot lead to optimality either but it certainly results in less complex subproblems.

Warner et al. [133], (1990), discuss patient-oriented and employee-oriented issues in nurse management. The latter are divided into several 'chronological' areas. The most interesting one is called short range scheduling and staffing. It includes the weekly, daily, or shift by shift adjustments to the long range schedule. The paper contains a description the history of computerised nurse scheduling in the US.

The article also introduces a nurse scheduling system called ANSOS (Automated Nurse Scheduling Office System) which will be discussed in more detail in Section 3. Warner et al. demonstrate a deep appreciation for the difficulties involved in tackling real world nurse rostering problems.

Bradley and Martin [22], (1990), distinguish three basic decisions in hospital personnel scheduling: staffing, personnel scheduling and allocation (as introduced by Warner [131]). The first problem consists of determining the long-term number of personnel that have to be employed. The number of personnel is expressed in terms of full time equivalents and is supposed to be sufficient to cover holiday periods (annual leave), training and further education. Hiring part-time nurses and allowing flexible work (or permitting the definition of different work agreements) facilitates a closer match between the personnel demands and the effective hours worked. Staffing decisions are influenced by the stochastic nature of personnel requirements and personnel capabilities. The second phase in Bradley and Martin's decision scheme is the conversion of the expected daily work force into precise assignments i.e. personnel rostering. Better schedules can be generated if the problem allows for differentiation between days of the week and seasonal variations. Since schedules are generated before the actual patient needs are known, the personnel manager or scheduler has to anticipate the personnel requirements. The third phase (allocation) consists of assigning the scheduled personnel to actual work sites. It enables the hospital to correct schedules as fluctuations in the demands occur. The time horizon for the allocation phase is typically very short (varying from a few hours to a couple of days).

Bradley and Martin present a useful classification of schedules both formally and from a solution method viewpoint, just like Sitompul and Randhawa [119] do. These can be summarised as: exact cyclical, heuristic cyclical, exact non-cyclical, and heuristic non-cyclical.

Siferd and Benton [117], (1992), present an excellent review of factors influencing hospital staffing and scheduling in the United States. A survey among hospital managers reveals the complexity of the problem. The work first discusses the staffing history in which cost reduction became more and more important. In the second part of the work, short-term personnel scheduling is discussed, in which various constraints on the nurses' schedules are taken into account. The researchers collected data from 31 different hospitals and, in total, 348 wards. Decentralised manual scheduling was the most common approach, often performed in co-operation with a large number of people per ward. The questioned hospitals worked with different skill categories for personnel. Personnel shortage is often solved by allowing overtime (sometimes leading to working days of 12 or 16 hours) and by using personnel from other wards. Full time work seemed to be more popular than any kind of part time work. It was also rather rare to have nurses doing both day and night shifts. In some cases the night work was carried out by a special group of personnel (but this does not generally hold). A large number of personnel is assigned to a set shift in practice. Most shifts have fixed start and end times. 50% of the hospitals work with three start times for day shifts on weekdays and 30% have 5 different start times. Most hospitals seem to work with stricter rules (e.g. in 93% of the cases, either there are no 'split' shifts, or, people are working the same days every week). This is an excellent paper which fully illustrates the difficulty of nurse rostering problems in practice.

Hung [63], (1995), collected 128 articles on nurse scheduling, from the 60's up until 1994, and presents an overview. Most papers study the experience of new work week arrangements. The overview is just a bibliographic selection but it can be useful for collecting literature from a variety of research domains. **Ernst et al.** [54], (2004), present a very comprehensive overview of the literature on staff scheduling and rostering. They have split the paper into three main parts:

- definitions, classification of personnel scheduling problems
- a classification of the literature into application areas
- solution methods, with comments on applicability.

Ernst et al. describe in detail what personnel scheduling and rostering involves. An extensive list of terminology and problem characteristics is introduced. They deliberately do not distinguish between rostering and scheduling. However, in a very interesting section about common decompositions of the problem, one of the proposals presents demand modelling as a separate module. Many of the characteristics apply to other problems than nurse rostering.

In the classification according to application areas, most attention is payed to crew scheduling and rostering. Ernst et al. consider that as the best covered application domain, due to the fact that it is economically more important than other areas. A large collection of papers is discussed. Mainly starting and finishing location issues distinguish crew scheduling from other personnel scheduling applications. Another type of personnel scheduling problem that is well covered in the paper is call centre scheduling. The complexity is caused by the workload that is never exactly known in advance. Many approaches therefore make use of queuing models and simulation techniques. The third category considers health care systems and it mainly involves nurse scheduling and rostering. Ernst et al. discuss, more or less in historical order, which approaches have been reported upon in the literature. Seven other application areas are introduced but they are less relevant from a nurse rostering point of view.

Also in the literature on personnel scheduling, mathematical programming and metaheuristic approaches are by far the most commonly applied techniques. The metaheuristics are promising for very difficult problems and for real world problems for which optimal solutions cannot be obtained with exact approaches.

Ernst el al. point out a few areas for improvement in the personnel scheduling and rostering domain:

- widening the applicability by generalising models and methods,
- gaining more efficiency through a better integration of the problem steps,
- increasing the accessibility by integrating the tools in ERP systems,
- catering for individual preferences,
- generalising the rostering algorithms.

There are a few PhD dissertations on the topic of hospital scheduling. Most of them belong to the staffing domain. They are briefly summarised in Appendix A.

It is clear that nurse scheduling problems have been addressed across a wide spectrum of research articles. It is also clear that some papers in the literature have tackled simplified problems and have developed methods which cannot be directly applied to hospitals. Even in the early days of automated nurse rostering [131], there was an awareness of other than just measurable optimisation objectives. Automated schedule generation needs to be flexible, to consider personal preferences, patient satisfaction, and human effort [119, 127]. In particular, Siferd and Benton started to show the way by highlighting the complexity of the real wolrd problem. Too many scientific articles in the past have not addressed this complexity. As we shall see, this is a theme which will recur in this paper and, indeed, it is one of our main conclusions.
The early papers on nurse scheduling research often considered the entire process of staffing, assigning people to wards, and short-term timetabling as a whole. The most recent overviews started to decompose the problem.

In Section 2.2, we will briefly discuss long term management decisions (which we call staffing). Staffing considers the management decisions that are made before the rostering process starts.

2.2 Staffing

Hospital staffing involves determining the number of personnel of the required skills in order to meet predicted requirements. It is also called workforce scheduling in other personnel planning environments such as production scheduling and cargo forwarding. In practice, several interrelated considerations make the task very complex [58]. Factors include the organisational structure and characteristics, personnel recruitment, skill categories of the personnel, working preferences [59], patient needs and circumstances in particular nursing units. Another significant staffing decision is to define work agreements for part time workers, to decide whether substitution of skill categories is allowed and for which people. In the real world problems discussed in this paper, staffing, budgeting and personnel rostering take place at different levels and for completely different time horizons. Many researchers have therefore decomposed the nurse rostering problem into phases (3 phases in [22, 131], 4 in [119], and 5 in [127]). Interaction between the levels is certainly necessary but in practice it would be unworkable to handle the problems simultaneously all the time. Personnel are usually hired for longer periods than the short-term rostering period. Although staffing and hospital management decisions are beyond the scope of this paper, a brief summary of some key articles is presented mainly because of the impact that different kinds of input data can have on the short-term nurse rostering problem.
Note that the literature overviews from Section 2.1 [22, 117, 119, 127, 131], nearly all concern staffing issues in addition to rostering.

As early as 1963, **Wolfe and Young** [138, 139], presented a model to minimise the cost for assigning nurses of different skill categories to various tasks.
In addition to dealing with rostering (discussed above), **Warner** [131], (1976), also deals with staffing in some detail. He defines the staffing problem as an annual decision in which seasonal variation can be considered. It consists of determining an appropriate number of full time equivalent nurses for each skill. A methodology for the staffing decision is proposed by Warner and many hospitals accept it (subject to small adaptations). After the scheduling phase comes the third step: the reallocation of nurses. This phase is a fine-tuning of staffing and scheduling. It involves determining how float nurses are assigned to units based on nonforecastable changes or absenteeism. Among hospital schedulers, the potential benefits of this reallocation step in the process are still uncertain. However, Warner is convinced that the combination of the three stages in the end leads to a better scheduling policy. Warner's paper is a key contribution to nurse staffing.
de Vries [48], (1987), developed a 'management control framework' to balance the supply and the demand for nursing care. There seems to be an acceptable range of balance between supply and demand instead of a strict equilibrium. de Vries calculates the actual capacity utilisation by dividing the workload per hour by the available staff per hour. Theoretically, uniform criteria could hold for all wards in the hospital. However, differences in workload between wards can be registered and result in a mechanism for co-ordination between wards.
The performance of the framework has been tested for real examples and the results are satisfactory. We believe that it is mainly due to the flexibility of setting parameters separately per ward, and to the expert knowledge on the floor that is used for forecasting the workload.
Smith-Daniels et al. [125], (1988), present a literature overview on capacity planning in healthcare. They

distinguish between capacity decisions on facility resources and on work-force resources. In these categories, two decision levels are selected: acquisition decisions and allocation decisions.
The acquisition decisions for work-force resources match the meaning of 'staffing' as it is defined in this section. Two other decisions in the group are the assignment of workers to units and to tasks. A useful contribution of the paper is the collection of many different strategies and approaches for the decision maker. Smith-Daniels et al. predict that the strict staffing and timetabling of people and other resources will all be combined in an objective for new large scale health organisations.
The allocation decisions for work-force resources, namely the assignment of workers to days and shifts, is not deeply studied in the paper. It may therefore be problematic to believe that the integration of all the planning categories into one objective will be applicable to complex real world problems.
Easton et al. [52], (1992), compare different staffing policies during a one month period in a large hospital in the United States. They are attempting to provide adequate staffing levels to meet the patients needs and attractive work schedules to satisfy the personnel. The research is carried out at the management level, considering costs and the annual percentage of personnel turnover, reflecting dissatisfaction. It is a common problem in hospital environments that unplanned capacity adjustments have to be made from time to time. In busy periods, unscheduled nurses will be expected to work, and in slack periods, people will work too few hours to earn their full wages. Restrictions on shift rotation and work stretches, distribution of unattractive work, higher wages for weekend and night work, 12-hour shifts during the weekend are considered. Alternative scheduling patterns (called ALTOURs in [52]) are becoming common in nurse scheduling environments. The patterns involve 8, 10, 12 or 16 hour shifts, combined with days off patterns and compensation days. Easton et al. also discuss the possibility of working with 'float' nurses. Float nurses can easily solve temporarily occurring under- and overstaffing in different wards. It is not recommendable however, to ask float nurses to undertake high risk tasks that require a lot of experience, such as working in intensive care and assisting in an operating theatre.
Finally, the paper presents the results of 12 different methods. It compares scheduled hour utilisation, paid hours, workforce distribution, and the number of different 'tours' (see also Section 2.4) for both unit scheduling and centralised scheduling (see Section 2.3). They conclude that the expected nursing expenses decrease as the scheduling alternatives increase. In order to obtain this result, the nursing requirements have to obey a number of rules that are explained in the paper. The research also excludes overtime, part-time work, and understaffing because it is very hard to formalise them. The results of [52] are thus quite problem specific. Due to different staffing and rostering policies in different hospitals, we believe that some of these extra parameters need to be taken into account in order to make the methods more widely applicable.

A few publications exist on simulation for personnel staffing in the emergency room [50, 74]. Although staffing policies are beyond the scope of pure nurse rostering, the rostering algorithms have to handle the results of management decisions at a high level. In some hospitals staffing and scheduling information is used to support structural change.

2.3 Administrative Modes of Operation

Different hospitals around the world have very different administrative procedures that lead to very different types of nurse rostering problems. We will briefly cover some of the main modes of operation in this section.
Centralised scheduling is a term that is sometimes used to describe the situation where one administrative department in a hospital carries out all the personnel scheduling [52, 117, 125, 131]. It relieves head nurses from the time consuming task of constructing schedules on a very regular basis. The major advantage of this approach is the opportunity for cost containment through better use of resources. However, centralised scheduling suffers from a number of limitations: personnel can feel that local ward requirements are not fed into the procedure, that the rosters are unfair, or even that there is favouritism (see [118] for more details).
When head nurses or unit managers are given the responsibility to generate the schedules locally, the process is sometimes called **unit scheduling** [5, 22, 49, 80, 84, 119].
Self-scheduling is a term sometimes used to describe the situation when the personnel roster is generated manually by the staff themselves (often with no or little computer aided support). The term 'interactive scheduling' is also used in this respect (e.g. Miller [87] and Ringl and Dotson [111]). Manual scheduling has been generally adopted in hospital wards. Self-scheduling is more time consuming than automatic scheduling but it has the advantage that the nurses co-operate and are asked for advice. Generally it is performed by the personnel

members themselves and co-ordinated by the head nurse of a ward. Nurses and other personnel collectively develop their schedules, taking coverage and time-related constraints into account. While the individual personnel members express their preferences for schedules and help setting the number of people required at any time, the personnel manager ensures that the hospital requirements are met. It is a very labour intensive procedure in which the nurses indicate their preferences and negotiate during breaks and before and after a shift. Some studies show that self-scheduling has significant drawbacks. In [118], for example, it is said that the process can easily lead to over- or understaffing, that the schedule is made for the convenience of staff, and that there are no formal procedures for conflict solving.

Miller [87], (1984), states that self-scheduling increases perceptions of autonomy among staff nurses, it reduces the head nurse's scheduling time, and it eliminates most of the special requests. The research is based on the implementation of self-scheduling in one single ward in a US hospital without comparing it with other nurse rostering environments. **Hung** [62], (1992), looked at more US hospitals and he identified a number of motivational benefits of self-scheduling. He states that it leads to greater staff satisfaction and commitment, it improves co-operation and team work and it reduces the staff turnover. Both authors found that self-scheduling yields more effective rosters because the personnel know that their personal preferences are taken into account and consequently they are more willing to co-operate.

Silvestro and Silvestro [118], 2000, discuss the results of a survey of nurse rostering practices in the UK National Health Service. They define the three different scheduling policies discussed in this section as follows:

- **Departmental rostering** which is conducted by the charge nurse or delegated to a staff nurse.
- **Team rostering**, where staff are divided into teams and a nominated member of each team has the responsibility for rostering, in consultation with team members.
- **Self-rostering**, where the roster is prepared by the ward staff.

In the practical nurse rostering problems that Silvestro and Silvestro studied, they identify 4 key determinants of rostering problem complexity:

- Ward size (measured by the number of staff).
- Predictability of demand (measured by the ratio of planned and emergency operations). This measure has not been studied in detail in our article because it is not part of the short term rostering problem.
- Demand variability (based on the variability in the length of patient stay and the degree of variation in the manning requirement over a working week).
- Complexity of skill mix (measured in terms of the degree of variation in staff grades and the complexity of the manning requirement specification).

For comparing the complexity of a wider variety of problems (Appendix B), we had to consider more measures than these four. Most of them are related to a more flexible shift type definition than the Early-Late-Night shifts discussed in [118].

Although certain papers in the literature advocate self-scheduling [87, 111], Silvestro and Silvestro conclude that the benefits and limitations depend on the operational context (ward size and rostering complexity). Departmental rostering seems more appropriate and effective for large wards with difficult rostering problems. Team rostering is suitable for medium sized wards with relatively easy problems, while self-rostering works well in small wards with relatively straightforward problems. Although self-rostering is more fashionable (according to the management literature [87, 111]), due to its empowerment and motivational benefits, there is a danger of generating unbalanced and inappropriate rosters.

2.4 Cyclical scheduling

In this section we will briefly discuss cyclical scheduling which concerns organisations in which each person works a cycle of n weeks [60]. This approach has some serious drawbacks for practical applications but it has received some discussion in the literature over the years. This type of schedule is common if the day is partitioned in distinct shifts and if the personnel requirements per day and per shift obey a cyclical pattern. Cyclical scheduling is also referred to as 'fixed' scheduling, while non-cyclical scheduling is sometimes called 'flexible' [118].

According to **Warner** [131], (1976) cyclical schedules offer several advantages. Personnel know their schedule a long time in advance, the same blocks are used repeatedly, the work is divided evenly, and unhealthy work rotations are avoided because it is common to apply 'forward' rotation. Forward rotation is met when a schedule includes no shift starting at an earlier time than a shift on the day before.

Megeath [79] proposed cyclical 7-day patterns of shifts and days off to allow for balanced shift coverage. There are clear benefits but cyclical schedules do not have high levels of flexibility. They cannot easily address flexible work regulations, fluctuating personnel demands and personal preferences. Also, cyclical scheduling requires a higher level decision to provide a precise number of skilled personnel members and strict personnel tasks. Working according to cyclical schedules is impossible if the problem is not very correctly stated.
Burns and Koop [37], (1987) developed a cyclical model for manpower scheduling with strict specifications on consecutive working days and days off. It takes only three different shift types into consideration. The model is not flexible enough to deal with complex modern nurse rostering problems.
Hung [61], (1991), presents a cyclical pattern for short-term nurse scheduling. He introduces 4-day workweeks with 10-hour shifts. Hung states that long shifts have benefits if the overlaps are strategically timed. Hospitals can cope with daily peak overloads, the communication between consecutive shifts is improved, and overtime is reduced. Hung allows 'downward' substitution in order to fill shortages for certain skill categories. The approach provides a permanent-shift system; this is a schedule in which nurses do not rotate. The advantages are that the people who work at the same time form a real team. There are also benefits for the social activities of the personnel members. For the scheduling problem, it requires the consideration of days on and off only, which reduces the complexity of the problem considerably. The constraints on the algorithm are coverage constraints and some time-related constraints: three free days per week and at least a number of free weekends per set of weeks. The algorithm is simple enough to be implemented manually. Some results are presented for problems in which the daily personnel requirements are not constant. The schedules match cyclical schedules to a high degree.
In [62] (1992), **Hung** identified the limitations of a cyclical approach. Cyclical schedules can only operate under the conditions of relatively stable demand and low variability, and they assume that sufficient workers can be recruited to staff the unpopular shifts. We believe that cyclical scheduling has many advantages but it is not directly applicable to many modern real nurse rostering situations. It might be a good idea to translate some of the benefits of cyclical scheduling into constraint based methods. Indeed, cyclical personnel rostering problems are generated using constraint satisfaction by **Muslija et al.** [94], (2000), and applied on real world examples (see also Section 3.3). The approach allows for the implementation of personal preferences but there is no evidence that it is applicable to large and complex problems.

Tour scheduling is a special case of cyclical scheduling. The tour scheduling problem is one of simultaneously determining days worked, start times and shift lengths worked over some planning horizon. By redefining traditional work weeks for nurses, many hospitals implemented new nurse schedules (or tours). As an example, alternative scheduling patterns (ALTOURs) were introduced in the work of Easton et al. [52] that has already been discussed in Section 2.2.
Bechtold and Showalter, [16], (1987) combine the problem of staffing and scheduling personnel in a tour scheduling model. A similar example of a tour scheduling approach is presented in [14]. Compared to flexible nurse rostering, the problems tackled with tour scheduling adopt many simplifications which restricts the opportunity for application in modern hospitals.

Although they can easily be generated and cover the personnel requirements, cyclical schedules are not flexible when it comes to addressing slight changes in personnel demands or in expressing personal preferences. Cyclical scheduling is only applicable in very rare cases. Moreover, personnel seem to prefer 'ad hoc' schedules. Such schedules address fluctuating hospital demands in addition to flexibility with respect to the private preferences of the personnel.

3 Nurse Rostering Approaches

This section will present and discuss the key approaches to the nurse rostering problem that have appeared in the scientific literature. The papers are grouped according to the type of method that is described.
Since the 1960's many papers have been published on various aspects of computerised healthcare personnel scheduling. Most mathematical scheduling approaches make use of an objective function which is optimised subject to certain constraints. Early papers [88, 129, 132, 134] are nearly all examples of optimising scheduling algorithms. Researchers attempted to develop linear models for the problem. When it comes to applications in practice, traditional mathematical algorithms have rarely been applied. As we have already established in Section 2, most nurse rostering problems are extremely complex and difficult. Tien and Kamiyama [127], for example, say that nurse rostering is more complex than the travelling salesman problem. For most real pro-

blems, the goal of finding the 'optimal' solution is not only completely infeasible, it is also largely meaningless. Hospital administrators want to quickly generate a high quality schedule that satisfies all hard constraints and as many of a wide range of soft constraints as possible.
Several heuristic methods have been developed for solving real world problems (e.g. [10, 20, 21, 66, 73, 110, 122, 123, 124]). These are discussed in some detail later on in this article. Wiers [136] discusses the applicability of operations research and artificial intelligence techniques for practical applications.
Although cyclical schedules are generally considered to be less difficult to generate, most of them are constructed with heuristic techniques. In the 1980's and later, artificial intelligence techniques for nurse scheduling (declarative approaches, constraint programming, expert systems) were investigated with some success. Some of these approaches are still relevant to today's research issues [39, 44, 85].
Many of the most recent papers (1990's and later) tackle the problem with meta-heuristic approaches. All of these approaches are discussed below.

3.1 Optimising Approaches: Mathematical Programming

Mathematical programming methods are appropriate for finding optimal solutions. However, their major limitation is that they are simply not appropriate for the enormous and complex search spaces that are represented by modern nurse rostering problems. Most researchers restrict the problem dimensions and consider a small set of constraints in their models. These models tend to be too simple to be taken into a real modern hospital situation.

Most of the mathematical approaches are based on optimising the value of a single objective function. A number of experiments have also been carried out with goal programming or multi objective decision making.

3.1.1 Linear and integer programming

Abernathy et al. [2], (1973), isolated nurse scheduling from the general staffing problem and solved it using mathematical (stochastic) programming techniques. They divide the staffing of hospitals into three decision levels: policy decisions (including the operating procedures for service centres and for the staff-control process), staff planning (including hiring, discharge, training and reallocation), and short-term scheduling of available personnel subject to the constraints imposed by the first two stages. Even the short-term scheduling in Abernathy et al.'s work involves more management decisions than the nurse rostering problem. The number of people required to fulfil the -stochastically varying- personnel demands is not pre-determined in Abernathy et al.'s paper. Specific skills have to be considered, unlike in other work environments where the quality of the work is less dependent on the person. The solution has an iterative and a non-iterative part.
The approach has only been applied to a hypothetical example application. Assignments of individual nurses to shifts happens on a day to day basis. We believe that this early paper's contribution is that it provides a very nice framework upon which future researches have posed more detailed and realistic problems.
Warner and Prawda [134], (1972), present a mixed-integer quadratic programming formulation to calculate the number of nurses from a certain skill category to do a number of shifts per day. Three non-overlapping shift types of 8 hours each are used. The goal function aims at minimising the difference between a given lower limit for the number of nurses and the variables, which are the number of nurses. By adding nursing time (i.e. employing more people), the cost for personnel shortage can be reduced (never under zero, however). The minimum staffing requirements should consider the possibility of replacing personnel members with different skills and the organisation's established standards.
In our opinion, this early approach cannot address the current needs of hospitals. There is no possibility of including personal preferences in the model. All the nurses are anonymous. An excess of nursing supply for a particular skill category can absorb (at some suitable rate) the shortage of other skills. A drawback of the approach is that an accurate forecast of personnel demand cannot be trustworthy for a period longer than four days.
Warner [132], (1976), elaborates on his previous formulation [134] by introducing weights or fairness levels. He works with shift patterns of 2 weeks length, with a fixed day and night rotation but which respond differently to some flexible constraints. Nurses and entire wards distribute a number of 'penalty weights' to constraints and thus to patterns and schedules. Certain parts of the scheduling are done manually before the optimisation starts: weekends are assigned by hand and there is also a manual determination of people who will rotate. This

simplifies the model. The mathematical programming algorithm consists of 2 phases: a search for feasibility and an improvement of the objective.
Even if the automated scheduling is not fully realised, this is a very interesting early contribution. One of the major benefits of the presented model is that it allows for a very fair evaluation of the obtained schedules. At the time of the publication (1976), the algorithm was implemented in several hospitals in the United States.
Trivedi and Warner [129], (1976), describe a branch and bound algorithm to arrange the short-term assignment of nurses from different units (called 'float' nurses) whenever there is a shortage of personnel. These mathematical approaches cope with small-scale problems only and as such are not really relevant to the modern nurse rostering environment. However, this is one of the first papers to specifically discuss the concept of float personnel and which has formally modelled the idea. The employment of floating staff still plays a major role in modern hospitals. There does not currently exist a methodology to deal effectively with floating staff and this is a possible area of future research. Our experience of implementing our system in over 40 hospitals in Belgium is that (at least in the Belgian case), float nurses quickly seem to 'settle down' in a ward with a temporary shortage of personnel instead of remaining available for the entire hospital.
Apart from presenting an interesting overview of automated nurse scheduling in the US, **Warner et al.** [133], (1990) present a nurse scheduling system called ANSOS. The system consists of four important modules:

- The Position Control Module maintains essential scheduling information for each employee (e.g. skills, types of shifts, weekends off/on, maximum work stretch and overtime start).
- The Scheduling Module uses information such as holiday requests and personal preferences, but also hospital-, unit-, and employee-specific rules to generate schedules. The module is based on a mathematical programming model and it produces schedules in a few minutes time. Interactive scheduling is carried out by adapting the weights and starting the scheduler again.
- The Staffing Module computes the required staffing levels for each unit, based on patient needs and sick calls, for example. The results are compared to the scheduled staffing levels. Inter-unit assignment changes can solve problems of over- and understaffing.
- The Management Reporting Module provides a large number of reports, which is essential for applicability in practice.

ANSOS takes a large number of individual constraints and preferences into account and it is applied to solving real problems in different hospitals (over 750 hospitals at the time of publication of [133]).
Miller et al. [88], (1976), formulate the personnel requirements in terms of minimum and preferred number of personnel per day, without specifying shifts. Staffing coverage and time related constraints with individual preferences are weighted against each other. Compared to Warner's approach [132], the number of unwanted shift patterns is much higher, thus reducing the complexity of the problem. The time related constraints are divided into two groups: the feasibility set and the non-binding constraints. Most of the non-binding constraints are stricter versions of already existing feasibility constraints. Apart from the objective penalty assigned to a violated constraint, an extra weight is added reflecting the nurse's personal perception of that violation. Miller et al. even introduce an 'aversion' index, which is a measure of how good or bad this particular nurse's schedules have been in the past. When granted, personal preferences cancel the violated binding constraints out. They reduce the number of possible patterns, and thus the search space, even further.
A cyclic coordinate descent algorithm is applied to look for a nearly optimal solution. A comparison with a branch and bound algorithm demonstrates that the algorithm by Miller et al. requires a much lower calculation time. However, the contribution of this early paper (in a modern context) is rather limited. Although the problems are very basic, the obtained solutions are not always feasible. Some schedules are under- or overstaffed on certain days of the planning period, which is what often happens in practice.
Bailey and Field [12], (1985), formulate a general mathematical model for the nurse scheduling problem. The cost function in their definition is the sum of the cost for utilising a shift type multiplied by the number of occurrences of that shift type in the schedule. Bailey and Field identify schedules minimising cumulative costs. Choosing one schedule out of the set can be done manually or by a linear program. Bailey and Field reduce idle time in schedules and they propose a 12-hour scheduling period instead of a traditional 8-hour period. Although the approach cannot address the current needs of today's hospitals, it is one of the few models that allow shifts to begin at any time during the day.
Rosenbloom and Goertzen [112], (1987), developed an integer programming algorithm for cyclical scheduling. The approach consists of 3 stages. A set of possible schedules is generated in the first stage. The resulting schedules are evaluated with respect to work regulations and work patterns. In the second stage, the minimum

daily coverage constraints are solved with an integer program. The third stage converts the solution into work patterns for each nurse. Even though optimal solutions are generated, the approach only considers work stretches and days off. In the modern hospital there is a large range of constraints and requirements which mean that the possibilities of directly applying the work described here are limited.
Jaumard et al. [69], (1998), propose an exact solution approach for a flexible realistic model of the nurse scheduling problem. The generalised linear programming model applies column generation and branch and bound. It allows full exploration of the set of feasible solutions. The authors claim that their model is more flexible to address changes in the scheduling environment than a heuristic model is. However, in practice, the conflicting nature of the nurse scheduling constraints makes it very difficult to find feasible solutions. The objectives include coverage constraints, salary costs, and care quality (balance between experienced and less experienced nurses). The approach is among the few that allow for formulating coverage constraints in terms of time intervals. However, a simplifying assumption of the model is that the demand periods are appropriately decomposed into smaller periods so that a given shift covers a given demand period entirely or not at all. This makes it less flexible than the model that is presented in [30]. Over- and understaffing is to be minimised but there is a pool of float nurses available when the coverage is not met. Preliminary tests have been carried out based on real data of a large hospital and the results are approved by head nurses.
Millar and Kiragu [86], (1998), combine all the possible 2-shift patterns of 4 days length for cyclic and non-cyclic nurse scheduling. They construct a network whereby each node represents a feasible pattern and they solve it with an algorithm based on the MIP solver in CPLEX. However, the proposed patterns do not deal with many of the requirements that are needed in large hospitals.

3.2 Goal programming/Multi-criteria Approaches

Mathematical programming techniques are not always flexible enough to cope with relative ranking assigned to various goals. They are often restricted to optimising one single goal or criterion. Goal programming defines a target level for each criterion and relative priorities to achieve these goals. The method aims at finding a solution which is as close as possible to each of the targets in the order of the priorities given. The approach is also called a 'multi-criteria' method.
Most of the papers discussed in this section apply mathematical programming but the latest modern research [19, 28, 68] tackles meta-heuristics in a multi-objective framework.

Arthur and Ravindran [11], (1981), propose a two phase goal programming heuristic for the nurse scheduling problem. They aim simultaneously to minimise (in priority sequence) staff size, the number of staff with ungranted requests or preferences, staff dissatisfaction, and the deviation between scheduled and desired staffing levels. A zero-one goal programming approach is used to assign days on and off to nurses. The shifts are heuristically assigned to the personnel members at the end of the scheduling process. Just like the mathematical approaches discussed in the previous section, the rather simple problem definition of [11] limits the scope of directly applying the method to the real world problems encountered in more recent publications.
Musa and Saxena [93], (1984), propose an interactive heuristic procedure for solving the nurse rostering problem. Users can change the relative weights given to the goals during the scheduling process, in order to take special temporal conditions into account. We believe that the latter feature is an important contribution. It serves the needs of modern hospitals because the scheduling circumstances change regularly and are very hard to model mathematically. With a two week planning period and one single shift to be scheduled, the complexity of the tackled problem remains rather low. The approach will not be directly applicable to most practical nurse rostering problems but it may well be worth investigating whether some of the ideas could be incorporated into modern methods.
Ozkarahan and Bailey [106], (1988), define three basic objective functions for their goal programming approach. The first goal is to minimise the deviation between the number of nurses scheduled and the demand, for each period of the day (called time-of-day scheduling). The second goal minimises the deviations between the sum of days on work patterns and the size of the work force (called day-of-week scheduling). With this goal, the system tries to schedule people according to their contract or work agreement. The third goal combines the day-of-week and time-of-day scheduling problems. Since the computational size of the studied problems is very large, they suggest the division of the work into two phases: one to determine schedules for the day-of-week and time-of-day schedules and one to assign people to the proposed schedules. By employing a heuristic

assignment of schedules, the algorithm solves the most important shift times and days for individual nurses.
Ozkarahan [102], (1991), presents a goal programming approach for a decision support system. The model aims at maximising the utilisation of full time personnel, minimising over- and understaffing, and minimising several kinds of personnel costs. It provides support for staffing decisions and for nurses' preferences. In comparison with the other problems discussed in this review (see the tables of comparison in Appendix B), the problem dimensions are very small. Ozkarahan (and Bailey) presented a very comprehensive linear model for nurse rostering (and partly also for staffing). The problem had to be decomposed in order to reduce the dimensions but applications are still limited to small size problems.
Franz et al. [55], (1989), developed a multi-objective integer linear program for health care staff working at different locations, called multi-clinic health regions. The problem involves staffing of personnel with varying skills, minimising travel costs and maximising the quality of service by considering personal preferences in addition to personnel requirements. Franz et al. developed a general optimisation approach, covering decisions at higher levels than short-term scheduling only. This contribution does not investigate the nurse rostering aspect in as much detail as other researchers do (see Appendix B).
Chen and Yeung [41], (1993), combine goal programming with expert systems. The assignment of shift types to personnel members is carried out by the expert system part of the approach (see also Section 3.3). Goal programming assists in satisfying the time related constraints on personal schedules and attempts to cover personnel demands in the meantime. This paper represents a particularly interesting contribution because it does not attempt to strictly capture all the problem characteristics but it allows for flexibility and the relaxation of constraints.
Berrada et al. [19], (1996), combine a multi-criteria approach for the nurse scheduling problem with tabu search (see also Section 3.4) in a flexible tool. In order to obtain a feasible solution, a set of hard constraints, related to administrative and union contract specifications, must be satisfied. An elaborate list of soft constraints is treated as a list of goals to be reached and the overall objective is to get as close as possible to these goals. Every nurse works the same shift all the time. This implies that the problem can be split into 3 single-shift problems of reduced complexity. Short-term requests of individual nurses are easier to implement and the scheduling problem remains simple. The approach produces satisfactory results after some tests on a few real problems. In order to really assist head nurses and save time, a software system should be developed in which it is possible to modify the weights for the different objectives.
This paper makes an interesting contribution because it is one of the first papers to abandon mathematical optimisation techniques in order to address a range of different goals.
Jaszkiewicz [68], (1997), introduces a decision support system for the nurse scheduling problem in Polish hospitals. Working days and free days are preferably grouped, the number of shift changes on consecutive working days should be minimised, and shifts have to be divided evenly among nurses. The problem is solved in two stages. In the first stage, a simulated annealing approach is applied in combination with a multi-objective algorithm (called Pareto-Simulated Annealing) in order to generate a set of good quality solutions. The samples are work stretches which meet the objectives in a satisfactory way. A hospital planner evaluates these results in an interactive way in the second phase.
It is unavoidable in practical nurse rostering environments to have an interactive planning module and Jaszkiewicsz has made that possible in his system and it is actually applied in a hospital. This makes the contribution an important one. However, the model takes many constraints for granted that, in our opinion, probably means that this system would require further work in order to be easily transferred to other hospitals. The Tables 13-20 in Appendix B, indicate that the work described in [68] appears among the less flexible approaches (with set values instead of user definable parameters).
Burke et al. [28], (2002), present a new multi-criteria approach after having implemented mainly cost function driven [29] meta-heuristics for nurse rostering in Belgian hospitals (see also [27, 31, 32] in Section 3.5). The criteria space, in which there is a criterion for each time-related constraint, is mapped to a preference space with dimensionless units. Previously developed heuristics [32] guide the search in this new solution space. One of the major advantages of this approach is the possibility for users to express their preference for certain constraints, instead of having to set very abstract cost parameters. The weights can control the compensation of constraints. A major advantage of the multi-criteria method is that it enables a better handling of dissimilar constraints by taking the possible ranges for the criteria into consideration. The method requires further development (to handle the full range of constraints required by hospitals) before it could be taken directly into a hospital ward.

Most mathematical approaches apply exact methods to find a feasible set of schedules. However, the real world problem is so complex that almost all the publications mention heuristic methods to assign work patterns to people and to cater for personal preferences needs.

3.3 AI Methods

3.3.1 Declarative and constraint programming

Okada and Okada [99], (1988), presented a formal core method in Prolog which assists in the assignment of shifts to nurses. The relative significance of various requirements can change during the planning period. Not all of the constraints have to be strictly satisfied. Okada and Okada find it even hard to define what an optimal schedule is. They distinguish between the scheduling task and the general requirements that must be fulfilled. The approach is much stricter than most others, the references appear in the 'set value' colums of Tables 13-20 in Appendix B. This paper represents a very systematic method to assign shifts whereas in approaches that are better suited for general application ([27, 84], for example), nearly anything can be scheduled and a penalty cost will be generated for the violated constraints. Assignments are carried out in a manual-like manner, following a strict procedure which is visible in the 'general requirements' above. **Okada** [98], (1992), elaborated on the general scheduling procedure presented in [99] in order to develop a system that can handle varying institution-specific requirements. A declarative program, generated through an interview with the user, models institution-specific information. The method represents a set of 'role sequences' as a language, in which the constraints are presented as a grammar. Individual preferences are then constraints on strings. It is, in our opinion, a very unique and interesting way to formulate the problem and it might be useful for setting up a formal description of nurse rostering problems. There are multiple criteria to evaluate the possible schedules for personnel members. By taking them all into account, the system tries to discover the 'best' schedules. The problem dimensions are comparable to those of [99] but there is an extra skill category, consisting of two leaders. Okada's system allows for a flexible definition of the soft constraints by the users of different types of hospitals.
Weil et al. [135], (1995), reduce the complexity of a constraint satisfaction problem by merging some constraints and by eliminating interchangeable values and thus reducing the domains. It is a sensible way of simplifying a problem and, although the generic model can cope with different legal regulations, Weil et al. still solve quite straightforward problems with it.
Darmoni et al. [45], (1995), describe a software system called Horoplan for scheduling nurses in a large hospital. Apart from rostering, the system also covers some short term staffing decisions. It is fit for use in practice but it does not really allow for very flexible problem definitions. Horoplan generates nursing schedules by applying a step by step procedure. It makes use of the constraint based programming tool called Charme. We assume that this sequence of steps reflects the way that head nurses create their schedules manually. Of course, the knowledge based rules and steps can never include the nurses' experience to deal with unsolvable problems and we believe that this is really necessary in hospitals today.
Meisels et al. [80], (1995), combine constraint networks and knowledge-based rules to solve employee timetabling problems. The described approach is implemented in a commercial software package, called TORANIT, which is particularly flexible with respect to defining constraints and shifts. It cannot guarantee optimal timetables because of the complexity of the allowed formulations. Compared to the constraint satisfaction approaches discussed above, we consider this as an advantage. Optimal schedules are of no avail anyway if the problems are dynamically changing as circumstances change. For the constraint programming approach, constraints fall into 3 groups:

- Mutual exclusion constraints: nurses can be assigned to one job at a time.
- Finite capacity of employees: for example: a limited number of daily/weekly/monthly working hours, a limited number of night shifts per employee.
- Objectives: constrain the distribution over time of employee assignments per shift.

The rule-based part of the system combines assignment rules and constraint rules, which are representations of human knowledge. Personal preferences for certain shifts are tackled by the assignment rules. This constraint orders assignments by preference, e.g. morning shifts preferred to late shifts. The constraint rules handle the demand for certain types of nurses or for individual nurses, in addition to personal constraints. Meisels et al. conclude that generic non-binary constraints in constraint networks and the ordering of constraints in line with their preference are very efficient to solve the constraint networks. **Meisels and Lusternik** [82], (1997) also investigate constraint networks for employee timetabling problems. Just like in [80], a flexible problem formulation should be possible. The approach consists mainly of standard constraint processing techniques, which solve randomly generated test problems. Experiments show that the domain size of the variables in the constraint networks is a critical factor for solvable problems. Such solutions do not necessarily exist in real life and constraint satisfaction techniques may not be the most convenient ones to handle infeasibility. Meisels and Lusternik also tested a genetic algorithm and found the same results.

Cheng et al. [43], (1997) developed a nurse rostering system for solving one particular hospital problem. They make use of the ILOG solver for generating a schedule that satisfies a large set of rules (such as preferred consecutive patterns and shift balance among nurses). The rules are divided into hard and soft constraints. Solutions are generated with a 4 step procedure, that is specifically oriented towards solving the particular problem (see also [42]). This contribution is less generic because of the problem specific modelling but nonetheless, it has been applied in a real modern hospital and as such is a significant relatively recent addition to the literature.

Meyer auf'm Hofe [84], (1997), presented the nurse rostering problem as a hierarchical constraint satisfaction problem. His research resulted in the development of a library containing various search algorithms and constraint propagation techniques. All of this is part of a nurse scheduling system (called ORBIS Dienstplan), which is tested on complex problems in hospitals and fire departments. The model provides the possibility of flexibly defining personnel requirements, provided that they are expressed in terms of shift types. It enables the use of arbitrary sets of shifts, by adding additional overlapping shift types to the traditional early, day and night shift. Generated schedules also have to meet requirements like: legal regulations, personnel costs, flexibility with respect to the actual expenditure of work, and the consideration of special qualities. Some of the previous considerations belong to a higher decision level than the pure short time rostering that is the subject of this review paper. It is not clear, however, how staffing decisions are implemented in the model. They might be also just input data. Work regulations determine to which extent requirements can be fulfilled, taking different skill categories into account. The software offers users the possibility of adapting it to their own needs. It enables the definition of work regulations and Meyer auf'm Hofe mentions qualified and experienced personnel but it is not clear how the system deals with them. The time related constraints are less general than in [29], for example, but they certainly belong to the most sophisticated of the published approaches. It is impossible in practice to satisfy all the constraints; Meyer auf'm Hofe therefore mentions 'partial' constraint satisfaction. Consequently, requirements are treated in order of importance. It is a very complex task to generate a satisfactory schedule in practical personnel planning situations, but the method is interactive and the user can alter the result of the algorithms by hand. The proposed approach is much more flexible and suited for real problems than many of the previously discussed papers in this review. In [85], (2000), **Meyer auf'm Hofe**, builds on his previous research and on experiences of the software system [84], which is used in practice. The generic model is developed for use in different real world personnel rostering settings. Hierarchy levels and constraint weights are defined. Instead of considering it as constraint satisfaction, nurse rostering should rather be looked at as constraint optimisation because it is not possible to satisfy all the constraints anyway. Fuzzy or 'non-crisp' constraints are introduced as constraints that can be partially violated and partially satisfied. A hybridisation of iterative improvement and branch and bound are used in a constraint propagation algorithm that deals with the fuzzy constraints. It is robust enough to handle varying formulations of the nurse rostering problem. We consider Meyer auf'm Hofe's work to represent a significant contribution to modern nurse rostering research.

Abdennadher and Schlenker [3, 4], (1999), present an interactive program that is based on constraint programming. The system has been tested in a real hospital environment.

Muslija et al. [94], (2000), tackle a different goal from the above described constraint satisfaction approaches. They attempt to generate cyclical solutions for a simplified version of general workforce scheduling problems. Rotating workforce schedules are beneficial for the employees' health and satisfaction, and thus increase the work performance of the personnel. Muslija et al. generate allowed (satisfying legal constraints) shift sequences in a one week planning period. Certain coverage levels must also be guaranteed. Important characteristics of schedules are the length of work blocks and 'optimal' weekend characteristics. Even when generating this type of rotating schedule, personal preferences and extra constraints can be implemented. The proposed method can assist in calculating good schedules very quickly but is probably too simplified to be of use in large scale healthcare environments.

3.3.2 Expert systems - Decision support systems

Decision support systems provide the possibility for developing user-interactive, integrated (staffing, rostering) approaches to nurse scheduling problems [97].

Smith et al. [122], (1979), developed a 'what-if' decision support system for various sets of weights instead of providing optimal solutions. A very nice feature of this early paper is that the software is interactive and that it allows users to assign weights to different objectives and to take personal preferences into account. The time related constraints are kept basic, which limits the system's applicability to modern general problems.
Bell et al. [17], (1986), developed a visual interactive decision support system for workforce scheduling. It is a rather basic solution but it is interesting because of the levels of interaction.
The decision support system introduced by **Ozkarahan** [102], (1991), makes use of a goal programming model (see also Section 3.1). The problem dimensions are kept very small. We can state that other problems discussed in this review (e.g. [27, 30, 32, 69, 80, 84]) would be far too complex to be tackled with the proposed method.
Ozkarahan and Bailey [106], (1988) describe three objectives in the goal programming approach (see again Section 3.1).
Chen and Yeung [41], (1993), schedule full time nurses with a hybrid expert system approach (see also [40]). The system handles constraints such as requested days off, maximum consecutive working days (restricted to 6), minimum consecutive free days, avoiding on/off patterns and minimising overtime. Some other fairness measures are also taken into consideration. In the meantime, the program attempts to meet minimum staff levels by applying a goal programming module. Unlike in many other approaches, where minimum coverage is a hard constraint, Chen and Yeung define aspiration levels for each goal. Minimum staffing requirements on particular days can thus be relaxed.
The expert system itself is involved in assigning early, late and night shifts. The other problem dimensions are very small compared to models that are also oriented towards assisting schedulers in practice. All the personnel members have the same work regulation and the same skill.
Scott and Simpson [114], (1998) reduce the global problem search space by incorporating constraint elements in a case-base. They attempt to generate good quality solutions within a limited time scale by mimicking the approaches of manual roster planners. Scott and Simpson identified the following steps in creating a case-base solution for the problem:

- Set up a case-base of sets of efficient shift patterns
- Analyse the (partial) problem and find the best case match
- Allocate shift patterns from the selected case to the nurses using an ordering rule
- Analyse shift allocation to find coverage problems
- Fix, if possible, coverage problems by legal swaps of shifts
- Store the solution as a generalised case if it is sufficiently different from existing cases.

There is no report on results, the paper only describes the main ideas of the approach (see also [113, 115]).
Beddoe et al. [108], (2002), present a case-based reasoning approach to nurse rostering problems. They attempt to automate the self-scheduling approach that is now applied in a particular UK hospital. Rules that have to be considered when constructing a schedule are: providing the appropriate skill mix, meeting the coverage, staff-to-patient ratios, and a set of time related constraints. Hospital planners combine and repair partial rosters that have been constructed from the personal preferences and requests. It is interesting that the methodology imitates the human style of reasoning in which problems are solved using past experience, on the premise that similar problems require similar solutions. The main ideas are comparable to those of [114] but this paper also provides results of tests carried out on real data and is in use in a modern hospital.

3.4 Heuristics

In practice, the size of nurse rostering problems, and the lack of knowledge about the structure of most of them, hinders the applicability of exact optimisation methods. Many heuristics have been developed to obtain high-quality schedules for real world problems in an acceptable computation time, without an explicit mathematical model.
The applicability of heuristic scheduling algorithms requires a clear formulation of the hospital requirements. It is necessary to quantify the quality of different schedules in an unambiguous way. Heuristic schedulers outline a number of steps in order to generate a schedule which respects the constraints (e.g. [110]). Most

heuristics are developed for generating cyclical schedules and they often emulate the trial-and-error manner whereby the planner constructs the schedule by hand. In this section, most of the contributions consider meta-heuristics, which represent some modern attempts to solve complex scheduling problems. **Smith** [121], (1976), presents an interactive algorithm which helps the scheduler to construct a cyclical schedule. The algorithm takes coverage constraints and days off policies into account and it determines the number of personnel members, which is a staffing decision. Not all the staff members can have rotating schedules, however. In view of more recent developments in nurse rostering, this early heuristic for cyclical schedules cannot really be considered for practical use today. **Smith and Wiggins** [124], (1977), developed a software system, using list-processing techniques that generate non-cyclical monthly schedules for several skill categories, which allows for different kinds of part time work. Schedules are developed per person, meeting the staffing requirements by alternating days off. This is more similar to short term nurse rostering than Smith's earlier work [121] and it also abandons the cyclical solutions. The model incorporates a considerable number of constraints: patterns, days off. It also allows the specification of the type of leave, which is an interesting feature for nurses that is rarely taken into consideration in the models that we reviewed. Another positive feature is the interactivity of the system. Users can make manual changes to the generated schedules.

Blau and Sear [21], (1983), generate all the possible shift patterns in a two week period. They evaluate them with respect to the nurses' preferences. A cyclic descent algorithm is used in the second step in order to find an optimal overall schedule with one of the 60 best patterns for each nurse. The approach is developed for wards with three skill categories in which substitutability is hierarchical. Both the problem and the model are comparable to those in [5, 49] (discussed in Section 3.5). However, Blau and Sear take over- and understaffing into account whereas Aickelin and Dowsland consider solutions with coverage deficiencies as infeasible. The contributions in [5, 49] are also concerned with achieving optimality. **Blau** [20], (1985), tries to equalise the distribution of unpopular work in addition to the frequency with which employees are granted requests for shifts or days. This is one of the earlier attempts (besides Warner's [132]) to evenly treat personnel with respect to workload and preferences. In later contributions (see also Table 18 in Appendix B), the distribution of work among people is often arranged via additional constraints.

Anzai and Miura [10], (1987), present a cyclic descent algorithm for a ward in which the personnel members are identical (with respect to skills and work regulations). Anzai and Miura state that their model is too simplified for practical applications.

Kostreva and Jennings [73], (1991), solve the nurse scheduling problem in two phases. Groups of feasible schedules are calculated in a first step. The groups respect the minimum staffing requirements and each individual schedule fulfils all major working constraints. In the second phase, the best possible 'aversion score', which is based on the preferences of the individual nurses [72], is calculated. The tackled problems are not complex. All the skill categories are scheduled independently, which comes down to a decomposition into partial problems that have hardly any connection with real modern practice.

Schaerf and Meisels [116], (1999), present a general definition of employee timetabling problems. It is the problem of assigning employees to tasks in shifts. The shifts are predefined time periods that can reside anywhere on the time axis. The model includes strict coverage constraints but it allows flexibility with respect to time related constraints. The problem involves exactly meeting the coverage and a set of time related constraints, while trying to meet preferences in assignments. A general local search is introduced that allows partial assignments and thus makes use of a larger search space. The paper presents hill climbing algorithms for the local search. Each technique concentrates on a different part of the search space, denoting their 'steepness'. In the approach, the neighbourhood functions can include 'insert', 'delete', and 'replace' moves. The approach has been tested in theoretical environments: a hospital and a production environment. However, the flexibility towards satisfying coverage (partial assignments), is a very interesting step towards solving problems in practice. It has been rarely applied before [30, 126] (see also Table 10 in Appendix B).

3.5 Meta-heuristic scheduling

We believe that meta-heuristics are generally better suited than other approaches for generating an acceptable solution in cases where the constraint load is extremely high and indeed in cases where even feasible solutions are very difficult (if not impossible) to find. Since infeasibilities are often unavoidable in today's practice, heuristics that cannot cope with them are of very little use.

3.5.1 Simulated Annealing

Isken and Hancock [66], (1990), belong to the rare group of researchers who allow variable starting times instead of 3 fixed shifts per day. They formulate the problem, which is (in other respects) rather simplified, as an integer program. Under- and overstaffing are allowed but penalised. We think that it is very relevant not to assume fixed shifts since many hospital activities are liable to flexible hours. Isken and Hancock thus solve a different problem than many other nurse rostering problems because of the flexibility in personnel coverage. Their model was never actually intended to address the nurse rostering problem. The model was a tactical tour scheduling model for addressing scheduling policy issues upstream from the short term timetabling problem [65].

Brusco and Jacobs [23], (1995), combine simulated annealing and a simple local search heuristic to generate cyclical schedules for continuously operating organisations. Apart from hospitals, many other organisations (including telecommunications, public safety, and transportation organisations ...) face demands for labour on a continuous basis - 24 hours per day, 7 days per week. Commonly, organisations that must service continuous demand allow their workers' schedules to begin at any hour of the week. This problem is rather exceptional and makes the scheduling process far more complex than shift type rostering.

The work concentrates on staffing by comparing the cost of alternative personnel scheduling options. Brusco and Jacobs call their problem a tour scheduling problem; it determines daily shift schedules and weekly days-off assignments for employees across a specified planning horizon. One of the most common flexibility alternatives for pure tour scheduling encountered in practical applications is the use of a mixture of both full-time and part-time workers (mixed workforce). One such approach involves a problem reduction that prohibits the use of daily shift schedules that would overlap from one 24-hour period to the next. The mathematical problem associated with this reduction is referred to as the 'discontinuous tour-scheduling' formulation.

Although the paper is more oriented towards staffing, we found it an interesting contribution for several reasons: it deals with part-time work, the assignments are not restricted to predefined shifts, and the approach can tackle different kinds of real problems that need not be solved to optimality.

3.5.2 Tabu Search

Berrada et al. [19], (1996) combine tabu search with a multi-objective approach (see also goal programming in Section 3.2). It is interesting that a meta-heuristic is applied instead of an optimisation approach but we think that the problem is of relatively low relevance to the real world situation because it looks only at switching days off and working days for different people.

Burke et al. [32], (1999) hybridise a tabu search approach with algorithms that are based upon human-inspired improvement techniques. Some of the hybridisations function as diversifications for the tabu search algorithm. A 'weekend step', for example, only attempts to solve some very specific soft constraints. Other hybridisation algorithms have been developed to provide a good quality roster that cannot be improved by small modifications. The details about changing the neighbourhoods during the search, and the benefits and limitations of a variable neighbourhood search approach, are explained in [31].

The resulting model is applied in several very different Belgian hospitals and thus it is suitable for problems with varying specifications. Users of the software system can define their own shift types, work regulations, skill categories and replacement strategies between skills (substitutability among skill categories is personalised in practice), in order to meet the hospital requirements. Coverage (defined as minimum and preferred coverage) is treated as a hard constraint but there are many optional procedures to assist hospital planners in setting them (see also De Causmaecker and Vanden Berghe [46]). While remaining in the feasible part of the solution space, the algorithms attempt to modify the roster in order to reduce the number of violations of time-related constraints on personal schedules. As is clear from Tables 13-20, the set of user definable constraints is very extensive. The time related constraints are divided into categories. Certain rules hold for the entire hospital. Underneath these global rules, each ward can define its own local rules. In Belgium there are many hospitals that allow for a personal work agreement per nurse. The presented model can handle that. Users can express personal preferences for certain shifts on certain days, and request days off easily. The model allows for selecting people who should preferably work together or preferably not. While evaluating the quality of a schedule, the previous planning period is also considered because there is an influence on the value of certain constraints. We believe that the nurse rostering problem tackled in [32] belongs to the most complex and flexible problems reported in the literature. Also, the model and algorithms are implemented in a system that has been used in over 40 Belgian hospitals. By automising the search for a solution, compared to a manual scheduling method, the approach of [32] reduces the calculation time and effort considerably and the resulting schedules are consistently better. An overview of our work in Belgian hospitals is presented in [33].

Dowsland [49], (1998), makes use of different neighbourhood search strategies in a tabu search algorithm. The heuristic oscillates between feasible solutions meeting the personnel requirements and schedules concentrating on the nurses' preferences. At any time of the planning period, the algorithm must provide enough personnel with the requested qualities, while satisfying the people by granting personal requests in a fair manner. The attractiveness of work patterns differs from person to person. Rather than designing a generic, widely applicable algorithm, the model was developed for solving the personnel scheduling problem in one particular hospital (see also Aickelin and Dowsland [5]). and produces very good quality results for that data.

The paper is interesting because it enables the search to go back and forth from the feasible to the infeasible region, whereas other approaches [19, 27, 32] tend to avoid infeasibility.

Valouxis and Housos [130], 2000, propose hybrid optimisation techniques for three-shift schedules. A list of feasible workstretches is enumerated by only looking at forward rotation. They integrate tabu search in an integer linear programming model. The complete ILP model would be inefficient but the performance is improved by applying an approximate ILP model. Better solutions are then generated by using local search techniques. In the neighbourhoods defined by the local search procedures, a tabu search algorithm is performed for finding high quality solutions.

Although there are more realistic problems in the literature, this approach takes some constraints into account that are particularly interesting for real nurse rostering problems. Examples are: balancing the workload among nurses, people can assign desirability to all the feasible workstretches.

Ikegami and Niwa [64], (2003), introduce a mathematical programming formulation for the nurse scheduling problem in Japan. They solve the problem with metaheuristics. A large survey of 315 units in 23 Japanese hospitals revealed interesting data for the problem description. In Japan, rapid shift rotation is quite common, i.e. shifts are changing several times per week and therefore most existing models are not applicable. The presented model covers features such as different skill categories, personal preferences, taking the results from the previous planning period into account, balancing the workload. These are all characteristics that seem unavoidable for real world applications.

Ikegami and Niwa distinguish between nurse constraints (which we call time related constraints) and shift constraints (called coverage constraints in this review). They decompose the problem into subproblems in which the nurses' schedules except one are fixed. The algorithms attempt to repeatedly satisfy the constraints on different subproblems. Algorithms are presented for 2-shift and 3-shift problems only, although the model can handle a variable number of shift types. A tabu search algorithm generates good quality results for the 2-shift problem in a reasonable amount of computation time. A branch-and-bound algorithm is presented for both the 2-shift and the 3-shift problem. It generates promising results but extra heuristics are required to speed the algorithm up.

Bellanti et al. [18], (2004), present algorithms for solving a particular problem in an Italian hospital ward. Tackling a real world problem, they deal with a large set of detailed constraints, which they divide into coverage constraints and contractual and operational requirements. Some unavoidable relaxations have been incorporated in the model, for example allowing a deviation between the coverage requirements and the actual number of scheduled nurses. The research has resulted in the development of a software system that is currently in use in the hospital. Its results improve upon those manually generated in the hospital and the computation time is satisfyingly low.

Bellanti et al. describe in detail how initial solutions are constructed. By applying different multistart procedures, different initial solutions are generated, from which the best is taken for the local search procedure. A tabu search method and an iterated local search approach have been developed. Both techniques outperform a multistart local search approach and iterated local search seems to perform slightly better than tabu search. Randomly generated test data can be requested for.

3.5.3 Genetic Algorithms

Easton and Mansour [51], (1991), developed a distributed genetic algorithm for an employee staffing and scheduling problem called 'tour scheduling'. The algorithm aims at minimising the number of personnel members to fulfil the demands. The fitness function represents violations of constraints and individual solutions are improved with local hill climbing operators. The genetic algorithm works very well for a set of test problems

but we think that the model is possibly too simple (e.g. no personal preferences) for real modern applications.
Tanomaru [126], (1995), presents a genetic algorithm to solve a staff scheduling problem. The objective is to minimise the total wage cost in a situation where the number of personnel is not fixed. Solutions have to meet the total workforce requirements while respecting the maximum number of individual working shifts. Overtime is allowed, however. Although the problem dimensions are very basic, this is one of the few research papers which allow flexible starting times for the shifts. Solutions for the personnel are represented by 7 pairs of integers, giving the start and stop times per day. For real-life problems, Tanomaru concludes that his heuristic mutation operators might be too time consuming. Also, we doubt whether the model is general enough for real problems. The number of constraints that are tackled is very low.
Aickelin presented a PhD thesis on Genetic Algorithms for Multiple-Choice Optimisation Problems (see Appendix A). One of the two problems he introduces to present his method is a nurse scheduling problem. The same problem is tackled by **Aickelin and Dowsland** in [5], (2000), where the evolutionary approach is a complex 'co-operative genetic algorithm'. Problem specific knowledge is used both to guide the crossover operator and a hill-climbing operator within the evolutionary algorithm. Separate soft constraints on the personnel schedules are not evaluated. Aickelin and Dowsland determine the value or penalty of weekly schedules beforehand (this is similar to what Warner [132] does). Only a limited number of such patterns exist and instead of evaluating constraints, the values per pattern are determined and saved. Nurses' preferences can change, so a cyclic schedule cannot be generated to satisfy the requirements. In the presented approach, Aickelin tries to decompose the problem in 'easier to solve' sub-problems. Night and day shifts are preferably not combined in a personnel member's weekly schedule. Night shifts can be scheduled separately to a certain extent. The skill categories are handled in a hierarchical manner in that higher qualified people can replace lower qualified people. This is an important contribution and it works very well for the personnel scheduling problem of a particular hospital. However, it is not applicable to the situation in all hospitals. Indeed in some practical situations it is undesirable to allocate highly qualified senior personnel to tasks for junior nurses. It is also the case that scheduling night shifts separately is not in accordance with some hospital customs.
The problem tackled by **Aickelin and White** [6], (2004), is exactly the same as the problem described in [5]. The constraints are not expressed explicitly but there is a set of feasible shift patterns from which the assignments have to be chosen. Two different algorithms are introduced: a genetic algorithm with an encoding that is based on an integer programming formulation that is presented in the paper; and an 'indirect' GA with a separate heuristic decoder function.
Aickelin and White propose a statistical method for comparing algorithms, even for algorithms that do not always generate feasible solutions on a chosen problem instance. The comparison algorithm provides a method to discover better parameter settings or components that improve the results. By applying the comparison procedure, a heuristic has been built that performs better than all the algorithms developed in previous research. The method can also be applied to other algorithms or to algorithms for other than the nurse rostering problem.
An evolutionary approach called a population-less co-operative genetic algorithm is applied to solve another 3-shift problem by **Jan et al.** [67], (2000). Feasible schedules satisfy the hard constraints, which are coverage constraints and personal requests for days off. The soft constraints are time-related constraints on personal schedules (a small subset of the soft constraints in Tables 13-20). A 15-days history of personal schedules is looked at for the evaluation. It is not clear how a feasible initial schedule is created. After the initialisation, the genetic algorithm searches solutions in the feasible region only. New schedules are generated by applying a two-point crossover on two personal schedules: the worst schedule and a randomly selected one. The search stops when a predefined number of generations is reached. Some optimisation methods have been explored: increasing the number of mates for crossover, diversification and the application of mutation and 'escape' operators.
In order to make this approach applicable to real hospital problems, more realistic individual cost functions are required, in addition to an evaluation procedure for the hospital planner to estimate the quality of a schedule.
Kawanaka et al. [71], (2001), propose a genetic algorithm for scheduling nurses under various constraints. A distinction is made between 'absolute' and 'desirable' constraints. Among the absolute constraints are the minimum coverage per skill category. The objective function considers weights for the desirable criteria: the balance of shifts, the granting of requested holidays, the number of night shifts assigned to unskilled, new nurses. Compared to a conventional method, which only implements the absolute constraints in the evaluation function, the presented approach generates considerably better results.
Applying crossover to nurse rostering nearly always causes problems of infeasibility (see also [27]). Kawanaka

et al. overcome this problem by exchanging shifts while attempting to maintain the characteristics of the parents. Unfortunately, they do not explain what these characteristics are and how they identify them.
In [27], (2001), **Burke et al.** elaborate on the problem described in [32] to address some of the shortcomings. They developed a set of genetic and memetic algorithms. Both types make use of a population of solutions but in the memetic approach, each of these individuals undergoes a local improvement before taking part in the recombination phase. The coverage constraints are satisfied throughout the search space. By simple recombination of two solutions it is nearly impossible to create a feasible solution and therefore repair procedures have been developed. The authors developed appropriate good quality schedule characteristics for inheritance to the next generation of solutions.
The solutions are relatively unaffected by initialisation and parameter changes. Thanks to evaluating a population of solutions instead of one single solution as in [32], the new approach overcomes inappropriate choices for the planning order of skill categories. The approaches are particularly robust to handle the variety of instances that occur in the real world. Although the memetic approaches generally lead to an increase in quality, the drawback is a higher computation time when compared to the results of a single population approach (see [31, 32]). Therefore, it is only applicable in practice when schedules are generated long before they are needed.

4 Conclusions

In this paper, we have briefly reviewed and discussed a wide range of nurse scheduling papers and articles that have addressed a broad spectrum of models, methods and approaches to the problem. Nurse scheduling has attracted the attention of scientists from Operational Research and Artificial Intelligence for about 40 years. The problem is of critical importance for a wide range of reasons. The automatic generation of high quality nurse schedules can lead to improvements in hospital resource efficiency, staff and patient safety, staff and patient satisfaction and administrative workload. Researchers have constructed and addressed a wide range of nurse scheduling models which have represented many different versions of the problem. Many techniques have been developed and described in the literature. Appendix B is a classification of the key methods that have been presented over the years and the aspects of the problem that they address. These techniques have varied from being quite simple to being very advanced. However, one key point that can be drawn from an analysis of the literature over the years is that very few of the developed approaches are suitable for directly solving difficult modern problems in practice. Many of the models that have been presented and discussed are too simple to be directly applied to hospital wards. This point is even more pronounced if we are to consider modern problems in large and busy hospitals. There is a definite gap between much of the current state of the art in nurse scheduling research and the demanding and challenging requirements of today's hospital environments. A crucial scientific goal for nurse scheduling research in the future is to fully address the needs and requirements of the real world. This goal has been addressed by several papers in the literature but there is still considerable scope for exploiting emerging technologies and providing the capability to develop a level of sophisticated decision support methodology that does not currently exist (see below). We illustrate the approaches that have been implemented in practice and those that have been tested on real world data in Table 1.

In the right hand column of Table 1, we illustrate the methods and approaches from the scientific literature that have been implemented in hospital wards. We have split these into those approaches that have been used in just one hospital and those that have been applied in more than one. This gives us some indication of the genericity of the methods/systems that have been applied in practice. The systems in the bottom right part of Table 1 (those that have been applied in multiple hospitals) represent the most flexible approaches that have appeared in the literature. The left hand column of Table 1 presents the papers that describe techniques which have been evaluated on real world data (but where there is no evidence that they have actually been used in practice). The nurse rostering approaches which do not address real problems and those which are only concerned with modelling issues are not included in this table. We have also not included those papers which deal with staffing issues (the stage before nurse rostering - see Section 2.2) because the focus of this paper is on rostering. It is important to note that we have constructed this table based upon the information available in the published papers. If this information needs updating then the authors should contact us. We intend to keep a regularly updated version of this table on the web. Authors and researchers with new information should contact us. The website is http://ingenieur.kahosl.be/vakgroep/it/nurse/applied_approaches.htm.

Not Applied in Practice but Tested on Real Data	Applied in Practice
Abernathy [2]	**Approaches applied in just one hospital**
Berrada et al. [19]	Easton et al. [52]: staffing
Beddoe et al. [108]	Jaszkiewicz [68]
Cheng et al. [42, 43]	Smith and Wiggins [124]
Okada and Okada [99], Okada [98]	Bellanti et al. [18]
de Vries [48]	**Approaches applied in multiple hospitals**
Warner and Prawda [134], Trivedi and Warner [129]	Warner [133]: ANSOS
Miller et al. [88]	Jelinek and Kavois [70]: Medicus
Muslija et al. [94]	Darmoni et al. [45]: Horoplan
Isken and Hancock [66]	Meisels et al. [81]: EasyStaff
Jaumard et al. [69]	Meisels et al. [80]: TORANIT
Aickelin and Dowsland [5], Dowsland [49]	Meyer auf'm Hofe [84]: ORBIS Dienstplan
Burke et al. [27, 28, 31]	Burke et al. [29, 30, 32], De Causmaecker and Vanden Berghe [46]: PLANE

Table 1: Applicability of the approach

Although we argue above that there is still a gap between theory and practice in nurse scheduling research, it is clear that the field has come a long way since the early days. One of the two main goals of this paper is to present an analysis and discussion of the key methods and approaches to nurse scheduling that have appeared over the years and to outline the current state of the art in the field. Table 1 clearly demonstrates that modern hybridised artificial intelligence and operations research techniques which incorporate problem specific information form the basis of most successful real world implementations. Exact methods represented some of the early approaches to solving the problem but they cannot cope with the enormous search spaces that are represented by real problems (at least on their own). There is evidence to suggest that there is research scope in hybridising exact methods with heuristic approaches. Although it is the case that exact methods may be able to aid heuristics it is clear that some kind of (hybrid) heuristic method offers the only realistic way of tackling such difficult and challenging problems in the foreseeable future. However, heuristic methods alone cannot cope with the dynamic and uncertain nature of modern problems (see below). The current state of the art is represented by interactive approaches which incorporate problem specific methods and heuristics (that are derived from specific knowledge of the problem and the required constraints) with powerful modern meta-heuristics, Artificial Intelligence approaches and constraint based methods.

The second main motivating goal of this paper is to present some future directions for nurse scheduling research that are based on a clear analysis of past scientific achievements and on a clear appreciation of the demands of environments in modern healthcare institutions. As already stated above, one obvious goal that modern nurse scheduling research has to tackle is to develop models and problem solving techniques that capture the needs and requirements of the real situation. As we have mentioned in the earlier discussion, many of the attempts to address nurse scheduling have tackled simplified problems. These simplified models may provide algorithmic insight (in a general sense) and they may provide benchmark problems to test new algorithms and developments. They serve a useful purpose within a general scientific framework. However, what we are arguing in this paper is that they do not directly advance our capability to solve real nurse scheduling problems in real hospitals. If this is our goal, then we must address the full range of requirements and demands that are presented by modern hospital workplaces.
This assertion leads to some challenging research issues which are constantly changing as emerging technologies impact upon the workplace. Here we list and briefly discuss some of the future research directions that we believe represent promising avenues for nurse scheduling research. Success in tackling some of these issues would make progress in closing the gap between nurse scheduling theory and practice. Such success would also underpin a new generation of more powerful decision support systems which better capture the needs of the hospital work place. Some of the important issues which need to be tackled and addressed are outlined as follows.

- **Multi-criteria Reasoning**: Nurse scheduling in hospital environments presents a range of objectives and requirements [33]. Many of these objectives are conflicting. For example, it is easy to see how an objective that aims to maximise staff preferences may conflict with an objective that requires a certain number of staff to work a certain shift. Multi-objective reasoning has been investigated successfully for a range of scheduling problems (e.g. [26, 77, 107]) and, indeed early steps to explore multi-objective reasoning have been made [28]. It can be argued, however, that because nurse scheduling is inherently multi-objective then nurse scheduling decision support technology should reflect this. It is clear that there is considerable scope for research in this area and it could be argued that multi-objective reasoning has a major role to play in future nurse scheduling research.
- **Flexibility and Dynamic Reasoning**: A key feature of real nurse scheduling problems is that the 'planned' personnel schedule usually has to be changed (often at very short notice) to deal with unforseen circumstances such as staff sickness and emergencies [30, 46]. As can be seen from our earlier discussion of the literature, very little work has been carried out on developing decision support technology that can address and deal with the uncertainty that is inherent in the real world. The question of how to develop such technology is very much an open question but it is one that the scientific community should address. Fuzzy reasoning has been investigated with some success [85] and there is a growing body of literature which deals with fuzzy reasoning for other scheduling problems (e.g. [120]). There is a certain amount of promise in investigating fuzzy reasoning as an attempt to address the dynamic nature of the problem in practice and as an attempt to deal with the inherent uncertainty that is a feature of the real world.
- **Robustness**: It follows from the previous point that the personnel schedule in hospital environments has to change to cope with unforseen events. We wish to avoid situations where (for example) just one person calling in sick causes a chain reaction of disruptions throughout the hospital because that person is the only scheduled person with a particular expertise (say). A more robust schedule would have had two (or more) people with that particular expertise scheduled to be at work at the same time. It is clear from the discussion in this article that robustness in nurse rostering has not played a major role in the scientific research literature. However, it does represent an important area in the uptake of nurse rostering research methods and it is important that the scientific community starts to address this issue with respect to real nurse rostering problems.
- **Ease of Use**: It follows from our earlier discussion, that many nurse scheduling research papers develop algorithmic approaches without any concern for the following question: If the developed algorithm were incorporated as the search 'engine' for a decision support system on a real hospital ward, how easy would the system be to use? As we have seen, academic research on nurse scheduling problems is often concerned with developing algorithms to solve various versions of the problem. The emphasis is on demonstrating that the developed algorithms are (in some sense) better than other algorithms. The usual definitions of *better* are that the algorithm produces lower penalty values on certain problems than other algorithms. Sometimes the amount of computational time required plays a role in the evaluation of nurse scheduling algorithms but it is quite rare for potential ease of use to play a role in these developments. Ease of use is, however, an absolutely crucial feature in the uptake of a decision support system. Many of the algorithmic methods that we have discussed in this paper require significant research expertise to employ (such as in setting the correct parameters). For example, it is unrealistic to expect a hospital administrator to alter cooling schedules for simulated annealing or the tabu tenure for tabu search or any of the other parameters that play a role in these and other meta-heuristics. Of course, there are ways to address this issue such as 'hiding' such parameter selection within the interface by employing easily accessible terminology or by pre-setting the parameters. However, a significant research goal for the scientific community would be to investigate algorithmic methods for nurse scheduling that are less reliant upon parameters. There may well be far more scope for implementing a 'parameterless' algorithm in the real world than for implementing a conventionally better algorithm (i.e. one which generates lower penalties on benchmark problems) which requires significant parameter tuning. It is also desirable to develop algorithms that have parameters that hospital administrators can understand and use. One such parameter might be the computational time that an algorithm is allowed. The research challenge would be to develop a method that can be presented with a variable amount of computational time and which intelligently uses all the time it has been given to effectively explore the search space. It is easy to envisage certain scenarios where an administrator needs an instant solution (e.g. when someone has just called in sick) and other scenarios where the user would like the algorithm to spend a lot of time to find a very high quality schedule (e.g. generating next month's

personnel schedule). Some work has been carried out in this area within the context of examination timetabling [25].

- **Human/Computer Interaction**: We know from the above discussion that scientific research into nurse scheduling problems has been primarily concerned with developing algorithms for various models of the problem. We have argued in this article that these models have often been too simple to directly apply to modern hospitals and we have also argued that the measures of evaluating algorithmic approaches do not consider some issues that are relevant to real world uptake (such as ease of use). The analysis of the current state of the art that is presented in this article shows us that it is also the case that little work has been carried out on algorithmic development for nurse scheduling that interfaces with human input to the scheduling process. However, if algorithms from the research community are going to be applied in real hospital situations then they have to interface with the administrators that will use them. This is a broad research direction which needs more consideration by researchers tackling nurse scheduling problems.
- **Problem Decomposition**: The idea of intelligently breaking up larger problems into smaller, easier to handle sub-problems and then dealing with each sub-problem in turn has been shown to work well on other scheduling/timetabling problems (e.g. [35, 38]). Of course, the major potential drawback with this type of approach is that it is possible to make allocations in earlier sub-problems which mean that later sub-problems cannot be solved. The key issue is to divide the larger problem into appropriate sub-problems in order to avoid this drawback and to develop appropriate mechanisms to attempt to avoid it as the allocations are made. A significant observation from the real world situation is that nurse scheduling problems can be very large and one way of dealing with large problems is to investigate decomposing them into smaller problems.
- **Exploitation of Problem Specific Information**: One of the key conclusions that can be drawn from this article is that the full range of constraints and requirements that are generated by real hospital problems has not been addressed enough in the scientific literature on nurse scheduling. We strongly argue in this article that these constraints and requirements should be investigated by the scientific community and that methods/knowledge/heuristics that deal with such constraints and requirements should be fully integrated with approaches to solve nurse scheduling problems.
- **Hybridisation**: It is absolutely clear from the discussions in this article and from the successes in other scheduling and optimisation applications that no one technique or method is going to increase the uptake of nurse scheduling research methods on its own. Progress will be made by drawing on the strengths and capabilities of a range of methods, approaches and research advances. The successfully applied approaches in Table 1 are all hybrid methods (in one sense or another). The research directions that have been briefly highlighted in this article are not single directions that should be explored in isolation from each other. On the contrary, each of these themes should be explored in conjunction with each other and successes in one should feed into the others and vice versa.
- **Inter-disciplinarity**: It can be seen from the articles that have been reviewed and discussed in this paper that they represent a broad range of 'traditional' academic disciplines. Nurse scheduling research has drawn on expertise in many disciplines in the past and we contend that, in the future, this inter-disciplinary aspect should not only be continued but should be actively and significantly increased. In our view, an inter-disciplinary approach to future nurse rostering research is absolutely crucial. We argue that increased inter-disciplinary collaborations between the scientific community represents a significant prospect for future research success. It can be easily seen that the research themes that have been outlined above will require research expertise from the disciplines of Operations Research, Artificial Intelligence, Healthcare Administration, Management, Psychology and Software Engineering (among others). We believe that the fostering of inter-disciplinary collaborations is one of the most important tasks that the scientific community should carry out in order to make serious scientific advances in nurse rostering research and to increase the uptake of that research in the real world.

It is important to note that the authors of this article completely recognise that the above (rather short) list does not represent all (or even close to all) the research directions that need exploring in nurse rostering research over the next few years. The list is simply an indication of just some of the directions that may lead to significant research advances in the area. It is also important to note that most of the above issues impact upon other scheduling problems but our arguments in this paper have been based exclusively upon the demands and needs of nurse rostering research. Of course, (as we have already mentioned) all of the above cannot be tackled in isolation from each other. All of these future directions impact upon each other to a greater or lesser extent. In particular, one absolutely critical issue that impacts upon all of these research directions is that they should be developed for and evaluated upon real world data. We believe that it is extremely important to start building up easily accessible benchmark problems from practice. It is also necessary that these problems include the issues that really are important to hospital administrators. We will start to collect such data and we would like to ask researchers in the community to help to contribute to this nurse scheduling problem archive. The data sets (together with instructions on how to contribute data sets) are available at http://ingenieur.kahosl.be/vakgroep/it/nurse/archive.htm.

In summary, there have been significant advances in nurse rostering research over the last forty years or so and there are significant research challenges ahead. We need to pay more attention, as a scientific community, to the issues that are important to modern hospital administrators in order to increase the uptake of nurse rostering research in the real world. Modern problems represent far more difficult problems than those that are tackled by many of the research papers in the literature. In particular, we need to be less rigid when we are evaluating algorithms. Of course, it is important to develop better penalty results on benchmarks and to work towards better computational time. However, features that are less easy to measure also play a role in real world uptake. These include levels of flexibility, ease of use, robustness, capability of handling uncertainty and human/machine interaction. It is very clear that the last 40 years of nurse rostering research has made significant advances but there are many more advances to be made and some very difficult research challenges to tackle. We look forward to seeing how they are addressed in the next 40 years!

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Appendix A
PhD dissertations

A list of PhD dissertations on the topic of hospital scheduling is summarised below:

- D.M. Warner: A Two Phase Model for Scheduling Nursing Personnel in a Hospital, Tulane University, New Orleans, LA, (unpublished), 1971
- D. Schneider: A Systems Analysis of Optimal Manpower Utilization in Health Maintenance Organizations, University of Florida, Gainesville, Florida, 1973
- V.M. Trivedi: Optimum Allocation of Float Nurses Using Head Nurses' Perspectives, University of Michigan, Ann Arbor, Michigan, (unpublished), 1974
- G. de Vries: Evenwicht in zorgvraag en zorgaanbod: besturing van de afstemming op verpleegkundigen (Equilibrium in Supply and Demand in Healthcare: Attunement on the Nursing Units), Eindhoven University of Technology, 1984
- M.V. Tobon Perez: An Integrated Methodology to Solve Staffing, Scheduling and Budgetting Problems in a Nursing Department, University of Pittsburgh, 1984
- D. Lukman: An Hierarchical Approach in Schedule Formulation and Maintenance under Uncertainty, University of Pittsburgh, 1986
  In the developed rule based decision support system there is no possibility for changing or adding rules, and the number of required personnel is not calculated. The system allows for qualitative considerations without quantitative values.
- I. Ozkarahan: A Flexible Nurse Scheduling Support System, Arizona State University, 1987
  A goal programming formulation, including both the determination of possible schedules and the assignment of individual nurses to these schedules is presented in this work. Ozkarahan realises that her formulation requires a zero-one integer program much larger than anything available at that time. The work is considered a part of a large decision support system which can incorporate artificial intelligence techniques in the nurse scheduling process.
- J.M.H. Vissers: Patient flow based allocation of hospital resources, Eindhoven University of Technology, 1994
  The research focuses on the analysis, design and control of operational health care processes and systems. Special interest areas are the development of the process concept and the allocation of shared resources within a hospital setting and beyond. The personnel scheduling part of this works belongs to the staffing domain.
- M.W. Isken: Personnel Scheduling Models for Hospital Ancillary Units, University of Michigan, 1995
  Isken developed several models and techniques for tactical scheduling analysis (denoted as 'staffing' in the review). The models consider all the specialised units in hospitals (laboratories, pharmacy) that are not regular wards. Results of Isken's models are the input for operational personnel scheduling.
- J.H. Oldenkamp: Quality in Fives: On the Analysis, Operationalization and Application of Nursing Schedule Quality, Rijksuniversiteit Groningen, 1996
  The thesis describes a study of the support of scheduling nurses, in which it focusses on the consequences of nursing schedules on the performance of the nursing unit. Three parts are distinguished in this performance: the effectiveness in providing nursing care, the efficiency of a nursing unit and, the influence of a nursing schedule on the nursing unit's performance.
- U. Aickelin: Genetic Algorithms for Multiple-Choice Optimisation Problems, European Business Management School University of Swansea, 1999
  Nurse rostering is one of the multiple choice problems of Aickelin's work. The corresponding part of that thesis is called 'A Direct Genetic Algorithm Approach for Nurse Scheduling'. It is the main aim to balance feasibility and solution cost or quality within a genetic algorithm framework.
- H. Meyer auf 'm Hofe: Kombinatorische Optimiering mit Constraintverfahren - Problemlösung ohne anwendungsspezifische Suchstrategien, University of Kaiserslautern, 2000
  The work builds on the development of a constraint library for a commercial personnel planning system. Soft constraints represent different kinds of restrictions on personal schedules, going from rather strict general conditions to flexible personal requests and costs. Search algorithms for the combinatorial problems make use of special propagation procedures from the constraint library.
- A. Ikegami: Nurse Scheduling in Japan -Modeling and Solution, Seikei University, Tokyo, 2001 (in Japanese)
- G. Vanden Berghe: An Advanced Model and Novel Meta-heuristic Solution Methods to Personnel Scheduling in Healthcare, University of Gent, 2002

Appendix B
A Classification of papers by the constraints and parameters that they consider

Tables 2 to 20 help to place each tackled problem within the context of the group of problems studied in the literature. They present a systematic overview of some relevant parameters and objectives collected from a range of publications. The authors are aware that the information in the tables may be incomplete and we ask other researchers to contact us with suggested changes. We will keep an updated archive of these tables at http://ingenieur.kahosl.be/vakgroep/it/nurse/classification.htm.
In the main body of the paper we were particularly concerned with the real world nature of the problems tackled: the flexibility of defining shift types, work regulations, skill categories, stochastically varying workload, the applicability in practice.

	Hard Constraints	**Soft Constraints**
Coverage	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]: allow relaxation of coverage constraints	Meyer auf'm Hofe [84, 85]
	Kawanaka et al. [71]	Chen and Yeung [41]
	Warner and Prawda [134]: minimum coverage is obligatory	Warner [132]: minimum coverage can be violated on predefined days
	Meisels et al. [80]	Warner et al. [133]
	Aickelin and Dowsland [5], Aickelin and White [6], Dowsland [49]	Okada [98], Okada and Okada [99]
	Schaerf and Meisels [116]	Miller et al. [88]
		Ikegami and Niwa [64]
Time Related Constraints	Berrada et al. [19]	Warner [132], Warner et al. [133]
	Miller et al. [88]: feasibility set (3 constraints)	Miller et al. [88]: non-binding constraints
	Bellanti et al. [18]	Burke et al. [27, 29, 30, 32], De Causmaecker and Vanden Berghe [46]
		Meisels et al. [80]
		Meyer auf'm Hofe [84, 85]
		Ikegami and Niwa [64]
		Aickelin and Dowsland [5], Aickelin and White [6], Dowsland [49]

Table 2: Coverage and Time Related Constraints are considered as Hard and/or Soft Constraints

When only considering the short-term rostering problem, two main goals are distinguished: coverage and time related constraints for personnel. It is necessary for the hospital management to provide enough assigned personnel at any time of the planning period, at the expense of violations on time related constraints. Table 2 groups papers according to the way they tackle coverage and time related constraints. Those who set coverage as a hard constraint do not allow any violations of coverage constraints at all. The upper left part of the table groups such papers. In other approaches, coverage constraints are preferably satisfied but they are treated as soft constraints (upper right part of Table 2). For very flexible rostering of real world problems, time related constraints are rarely hard. However, two examples of approaches that do not accept violations of such constraints are given in the lower left part of the table. Most models attempt to minimise violations on time related constraints that are treated as soft constraints. These are grouped in the lower right part of Table 2.
De Causmaecker and Vanden Berghe [46] present a set of relaxation procedures for the coverage constraints.
The approaches in which personnel requirements are not hard allow management level decisions in the short-term planning. These that have hard time related constraints all have fewer and less strict constraint types.

Objectives	**Optimising**	**Heuristic**
Minimise violations of time related constraints	Warner [132]: schedules constructed with predefined patterns, the objective is to minimise $\sum_{people}$ ('aversion' for the pattern)	Burke et al. [29], minimise $\sum_{people}$ (violations on soft constraints), see also [27, 31, 32], De Causmaecker and Vanden Berghe [46]
	Warner et al. [133]	Arthur and Ravindran [11]: minimise staff dissatisfaction by minimising the number of staff with ungranted requests
		Aickelin and Dowsland [5], Aickelin and White [6], Dowsland [49]
Minimise violations of coverage and time related constraints	Miller et al. [88]: nearly optimal solution generated with a mathematical algorithm	Okada [98], Okada and Okada [99]
		Ikegami and Niwa [64]
Minimise violations of coverage constraints		Bellanti et al. [18]
Minimise number of employees	Alfares [8]	Arthur and Ravindran [11]
		Easton and Mansour [51]
Minimise personnel cost	Jaumard et al. [69] satisfy the coverage while minimising the salary costs and maximising the employee preferences and team balance	Meyer auf'm Hofe [84] takes personnel costs into account in addition to the cost for expenditure of work
	Tanomaru [126]	
Minimise non-negative 'under' coverage	Warner and Prawda [134]: the cost for 'nursing care shortage' is minimised	
Uniform distribution of shortages and surpluses over weekdays	Berrada et al. [19]	
Minimise deviation between scheduled nurses and demand		Ozkarahan [102]: minimise nurse shortages and surpluses
		Arthur and Ravindran [11]
Minimise deviation between scheduled people and the total work capacity from the work regulations	Ozkarahan and Bailey [106]	

Table 3: A summary of which approaches, Optimising and Heuristic, tackle which objectives

The objectives differ from approach to approach, which is clear from the straightforward categories in Table 2. In Table 3, a list of possible goals is presented in two different categories: the optimising and the heuristic approaches. Examples of objectives are:

- Minimise 'under'coverage. The coverage constraints are soft constraints.
- Minimise deviation between scheduled nurses and demand. In this approach both over- and undercoverage are allowed but not wanted, the deviation from the coverage constraints has to be minimised.
- Minimise number of employees. The number of employees is not fixed and this objective belongs more to the staffing domain.
- Minimise personnel cost. Labour costs can be reduced by minimising the amount of overtime but also by minimising the number of expensive staff.
- Uniform distribution of shortages and surpluses over weekdays. Violations of the coverage constraints have to be evenly spread over the days of the planning period.
- Minimise deviation between scheduled people and the total work capacity for the work regulations. The

purpose of this objective is to satisfy the contractual requirements better and thus reduce over- and undertime.

Parameters	Constraints	Costs and Weights
Fixed: predefined parameter values and constraints	Aickelin and Dowsland [5], Aickelin and White [6], Dowsland [49]	Berrada et al. [19]: weights are fixed
	Warner [132]	
Adaptable: users can set the values of the problem parameters	Musa and Saxena [93]	Musa and Saxena [93]
	Warner and Prawda [134]: a few organisational constraints	Warner [132]: personal and unit wide 'aversion' for patterns
	Miller et al. [88]	Miller et al. [88]: personal 'aversion' for non-binding constraints
	Ikegami and Niwa [64]	
	Bellanti et al. [18]	
	Okada [98]	
User Definable: the users define the problem parameters	Burke et al. [27, 28, 29, 31, 32] and De Causmaecker and Vanden Berghe [46]	Burke et al. [27, 29, 31, 32] and De Causmaecker and Vanden Berghe [46]
	Weil et al. [135]: generic model can cope with different legal regulations	weights in Burke et al. [28]
	Meyer auf'm Hofe [85]	Meyer auf'm Hofe [85]
	Meisels et al. [80]	Meisels et al. [80]

Table 4: Flexibility of setting and defining problem parameters

Examples with fixed constraints and parameters are quite rare and they are very often pure theoretical implementations of one single problem. Most models allow the user to adapt some predefined constraints and penalty values to their own needs (Table 4). The column Constraints denotes how flexible the models are towards defining constraints. Higher up in the column are the approaches with fixed constraints. The second level in the column shows contributions in which constraint parameters can be set by the users. The most flexible ones are at the bottom. They are models in which users can define the problem parameters themselves. We have collected information about modifiable costs and weights in the right column. Again, the papers are presented in order of increasing complexity towards the bottom of the table.
Flexible software systems, which are extendible with new constraints and which have modifiable costs and weights, are, of course much more difficult to build.

Generally, the design of cyclical schedules requires more than short-term rostering decisions only (see also Section 2.4). However, once the requirements are set, cyclical schedules are much easier to generate than others because the search space is considerably smaller.
Table 5 divides approaches into cyclical, semi-cyclical, and non-cyclical methods. The left column enumerates some of the pure cyclical methods. The models that are grouped in the middle column describe mixed approaches. Burke et al. [29], for example, developed an intrinsically non-cyclical model but it provides means for setting constraints that superimpose cyclical patterns upon certain personal schedules. The model thus includes certain cyclical aspects for non-cyclical rosters. Warner [132] proposes a different solution in which personal rosters either obey cyclical or non-cyclical rules.
The last column groups models that are non-cyclical. They either allow all the possible variations of schedules and penalise the bad ones, or they select schedules from a limited set of desired shift sequences.

Many researchers are aware of the necessity of small violations of the coverage constraints when it comes to scheduling in practice (see Table 6), and penalise them in a cost function. The approaches that are grouped in the first column (where understaffing is allowed) and in the third one (overstaffing is allowed), take coverage as a goal to be reached. The contribution of [46] is different. The coverage constraints are expected to be satisfied. If they are not carefully defined by the users, a consistency check will indicate infeasibilities. The

Cyclical	Semi-Cyclical	Non-cyclical
Chan and Weil [39]: but flexible with respect to annual leave and unexpected events	the model in Burke et al. [29] provides the possibility to define cyclical patterns that can be superimposed on non-cyclical schedules	Burke et al. [27, 29, 28, 32], De Causmaecker and Vanden Berghe [46]
Muslija et al. [94]	Warner [132]: manual preprocessing of the number of people who rotate day and night weeks	Aickelin and Dowsland [5], Aickelin and White [6], Dowsland [49] (however, night shifts cannot be scheduled together with morning/late shifts during the same week)
Alfares [8]	Smith [121]: not all the personnel members have a rotating schedule	Okada [98], Okada and Okada [99]
Burns [36]	Chan and Weil [39]	Miller et al. [88]
Burns and Koop [37]		Meyer auf'm Hofe [84, 85]
Baker [13]		Kawanaka et al. [71]
		Schaerf and Meisels [116]
		Ikegami and Niwa [64]

Table 5: Summary of Cyclical and Non-cyclical Approaches

model in [46] provides several planning options to find the best coverage in every situation. The last column of the table presents some of the more flexible models with respect to coverage. They compare best to the way coverage is treated in real hospital environments.

We noticed that there is a large gap between solving real problems and solving theoretical ones. Most authors restrict the applicability of their models to some simplified examples of nurse rostering, with, for example, three different shifts, short planning horizons, a limited number of possible patterns for personnel members with an identical work regulation. In Tables 7 to 12, we have grouped specific dimensional units in order to identify the difficulty of the problems addressed.
We say that skill categories are hierarchically substitutable when higher skill categories can do jobs replacing lower skilled people. In other situations, people from different skill categories can substitute each other in a user defined way. The latter approach better reflects the situation in hospitals. Among the group of people belonging to the same skill category, some are more experienced or have better management skills to replace the head of their department.
In Table 7, we have grouped the publications according to the number of skill categories that the models can handle (columns), and to the way substitutability is organised. The leftmost columns contain the simpler models, whereas the columns on the right include models that are fit for practical applications. The top-down evolution in the tables also goes from simpler towards more complex models.

In simplified research examples, nurse rostering problems can be defined with equal constraints for all the personnel members. The assignment of schedules to people is in that case very straightforward. More realistic examples take part time contracts into account and provide flexibility to define personal work agreements. We have made a classification (Table 8) of contributions according to the way they handle work regulations or contracts for different personnel members. The upper part of the table shows which approaches treat all the nurses equally with respect to their contract (involving the time related constraints). It generally means that all the personnel members have a full time contract. The second part of the table refers to mixed approaches in which full time and half time contracts are combined. This is probably still not applicable to most real problems. Models that allow the users to set all the parameters related to contracts and personal constraints are grouped in the next part of the table. They are all more oriented towards applicability in real hospitals. The lowest level of the table shows the papers that mention 'float' nurses. It often occurs that temporary personnel shortage is solved by people from another ward. The references in this table are thus organised from top to bottom in groups of increasing complexity.

Understaffing		Overstaffing		More Options
Allowed	Not Allowed	Allowed	Not Allowed	
Miller et al. [88]	De Causmaecker and Vanden Berghe [46]: unless certain circumstances occur, consistency check	Miller et al. [88]	De Causmaecker and Vanden Berghe [46]: unless certain circumstances occur (e.g. avoiding undertime	De Causmaecker and Vanden Berghe [46]: consistency check, the range between minimum and preferred coverage is flexible
Warner et al. [133]	Burke et al. [27, 29, 31, 32]	Warner et. al [133]	Burke et al. [27, 29, 31, 32]	
Warner [132]	Warner and Prawda [134]	Warner and Prawda [134]		
Ozkarahan [102]	Kawanaka et al. [71]	Ozkarahan [102]		
Isken and Hancock [66]	Meyer auf'm Hofe [85]	Isken and Hancock [66]		Meyer auf'm Hofe [85] defines minimum and standard staffing levels which are treated as fuzzy constraints, there is a considerably larger penalty for understaffing than for overstaffing
Bellanti et al. [18]	Jan et al. [67]	Jan et al. [67] only for day shifts		
	Schaerf and Meisels [116]	Okada [98], Okada and Okada [99]	Schaerf and Meisels [116]	
	Valouxis and Housos [130]	Valouxis and Housos [130]		
	Aickelin and Dowsland [5], Aickelin and White [6], Dowsland [49]		Aickelin and Dowsland [5], Aickelin and White [6], Dowsland [49]	

Table 6: Approaches for tackling Coverage Constraints

It is also shown in Table 8 that in case of personnel shortage, many hospitals make use of a group of 'float' nurses, to assist temporarily.

Most researchers are aware of regular changes in coverage constraints. This is one of the reasons why pure cyclical schedules are generally not workable. In Table 9, we have grouped the approaches in categories of flexibility. The top of the table shows the least flexible papers with respect to changes. Warner and Prawda [134] predict the personnel requirements for the next few days. The personnel requirements are nearly always expressed as the number of people required per shift type or even per day.
Modelling very detailed, hardly varying coverage constraints generally causes no problem for generating solutions. Miller et al. [88], Burke et al. [30], and Meyer auf'm Hofe [84, 85] are among those that leave a large part of initiative to the schedulers. Burke et al. [30] tackle the problem in a flexible way, as a result of feedback from the users of their software system in several hospitals. Not only is the number of possible shift types higher than in most problems encountered, but also the approach to compose a schedule with different combinations of shift types is exceptional.
The last column of the table groups papers in which the coverage constraints are defined as a range between minimum and maximum coverage. Such a range is more useful in practice than very strict numbers.

	Number of Skill Categories			
	1	**2**	**3**	**User Definable**
Skill Categories Scheduled Separately	Weil et al. [135]	Ozkarahan [102]	Musa and Saxena [93]	Schaerf and Meisels [116]
	Chen and Yeung [41]	Okada and Okada [99]	Arthur and Ravindran [11]	Ikegami and Niwa [64]: nurses are divided in skill groups
	Isken and Hancock [66]		Kawanaka et al. [71]	
	Bellanti et al. [18]		Warner [132]	
			Okada [98]	
Hierarchical Substitutability: people with a higher skill category can replace the lower			Hung [61]	Meisels et al. [80]
			Aickelin and Dowsland [5], Aickelin and White [6], Dowsland [49]	
User Definable Substitutability: some nurses can replace people from another skill category, it depends on their qualification			Miller et al. [88]: a subgroup of the regular nurses might be the group of those who can perform as head nurses	Warner and Prawda [134]: small overlap (substitution) between skill categories allowed, not related to people individually
				Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]

Table 7: Number of Skill Categories and Substitutability

Some approaches generate schedules that only consist of days off and on. The next step in the process, the assignment of actual shifts to people, is left for a head nurse to do manually. These papers are grouped in the upper part of Table 10. The possible shifts are strictly distinct since there is no overlapping between an assignment and a free day. It is very common in nurse rostering to consider three different shift types (the second group in Table 10). We found many examples of models with three strictly distinct shift types but also some that allow an overlap in time between the shifts. However, the latter are not more flexible because they have set start and end times for all the three shift types.
Algorithms developed for use in practical healthcare environments do not work with three strictly distinct shift types (see the lower half of Table 10). The activities in hospitals are so varied that a large number of user-definable shifts is allowed. In [30] start and end times can even be personal for some practical reasons as a result of a negotiation with the hospital manager. The higher the number of shift types, and the more flexible they are, the larger the search space is.

Short planning periods are much easier to generate schedules for. It can be an option to split the period into smaller intervals and to combine the schedules afterwards. This will lead to sub-optimal solutions in nearly all the cases. Table 11 gives examples of some of the realistic and theoretical approaches studied in this article. The first example [134], has the shortest planning period of all. Forecasting the demands is not possible for a period longer than 4 days in that model. The short planning period leads to a rather simple rostering problem. However, it is not a very good idea not to fix the coverage constraints long beforehand because that does not correspond to the needs in real hospitals.
The next rows in Table 11 present other approaches with fixed and semi-fixed planning periods. More complex models, with longer planning periods, are listed lower down in the table.

The number of personnel members in a hospital ward can vary from less than 10 to well over 100. In the

Identical Work Regulations for all People	Miller et al. [88]: full time nurses only
	Arthur and Ravindran [11]: full time nurses only
	Weil et al. [135]: full time nurses only
	Warner and Prawda [134]: no distinction between people
	Chen and Yeung [41]
	Bellanti et al. [18]
Mixed Workforce: Full Time and Half Time nurses	Ozkarahan and Bailey [106]: different work regulations
	Musa and Saxena [93]: various part time options are possible
User Definable Work Regulations	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]
	Meyer auf'm Hofe [84, 85]
	Chiarandini et al. [44]
	Schaerf and Meisels [116]
	Warner [132]: people can have different 'contracted workloads'
Float Nurses: nurses from different wards can solve personnel shortage	Warner and Prawda [134]: generally, nurses are assigned to a unit and do not move around at zero cost; a few 'float' nurses do move around
	De Causmaecker and Vanden Berghe [46]: it is possible to let people work in more than one ward
	Meyer auf'm Hofe [84, 85] constrains the expenditure of work
	Warner et al. [133]: a special 'float pool' of personnel is scheduled to facilitate last-minute changes
	Trivedi and Warner [129]

Table 8: Flexibility in setting and defining Work Regulations

	Days	**Shifts**	**Hours**	**Minimum-preferred**
Constant		Jan et al. [67]		
Weekdays-weekends		Kawanaka et al. [71]		
Fluctuating: the coverage constraints vary over the planning period	Miller et al. [88]	Warner [132] Warner et al. [133]	Miller et al. [88]	De Causmaecker and Vanden Berghe [46]
	Alfares [8]	Burke et al. [27, 32]	Burke et al. [30]	Burke et al. [27, 30, 32]
	Meyer auf'm Hofe [84, 85]	Warner and Prawda [134] (4 days ahead)	Meyer auf'm Hofe [84, 85]	Warner and Prawda [134]
		Aickelin and Dowsland [5], Aickelin and White [6], Dowsland [49]		Ikegami and Niwa [64]: lower and upper bound
		Bellanti et al. [18]		

Table 9: Different definitions of Coverage Constraints

	Strictly Distinct	Overlapping Allowed
1 Single Shift or no Shifts Defined: Days	Miller et al. [88]	
	Musa and Saxena [93]	
	Narasimhan [96]	
3 Different Shifts: usually referred to as Morning, Late, and Night shift	Warner [132]	
	Trivedi and Warner [129]	
	Aickelin and Dowsland [5], Aickelin and White [6], Dowsland [49]: the night shifts are scheduled separately to a certain extent and thus the complexity is reduced to a 2-shift problem	Okada [98], Okada and Okada [99]: the shifts have very strict start and end times: a different morning shift (same start time, half the duration) is accepted on Saturdays
	Warner and Prawda [134]: 3 8-hour shifts per day	Hung [61]: 3 slightly overlapping 10-hour shifts
	Berrada et al. [19]: there is no rotation: the problem can be split into 3 single-shift problems	Weil et al. [135]: 8-hour day and evening shifts and 10-hour night shift
	Bellanti et al. [18]	
	Jan et al. [67]	
	Strict Start-End Times	**Floating Intervals**
Defined Length: the intervals cannot be set by the users		Bailey and Field [12]: 12-hour shifts instead of 8-hour shifts; the shifts can start at any time of the day
User Definable Shifts: the number of different shifts, their start and end times and their length are set by the users	Meyer auf'm Hofe [84, 85]	Tanomaru [126]
	Meisels and Lusternik [82]	Brusco and Jacobs [23]
	Schaerf and Meisels [116]	Schaerf and Meisels [116]: allow partial assignments
	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]	Burke et al. [30]
	Chiarandini et al. [44]	
	Kawanaka [71]	Isken and Hancock [66]: variable starting times instead of 3 fixed shifts per day
	Meisels et al. [80]	Jaumard et al. [69]: assumes demand periods that correspond to the shift times

Table 10: Flexibility of defining Shift Types

cases where the problem cannot be split into sub-problems, the algorithms must be powerful enough to solve problems with widely varying parameters. Table 12 presents an overview of the models with respect to staff size. The upper half of the table shows a list of models in which the user can set the number of nurses to be scheduled. The lower part groups problems that attempt to minimise the staff size in a ward. In the classification of scheduling problems, setting the number of personnel rather belongs to staffing than to nurse rostering.

Tables 13 to 20 present a list of time related constraints. Some researchers set strict values for the constraints, whereas others let them be user definable. A few authors apply constraint programming techniques in their model (e.g. Meyer auf'm Hofe [84, 85], Meisels et al. [80]). Others have set up a new formal evaluation method for a very extensive set of modifiable timerelated constraints (Burke et al. [29]). If we compare the tables listing the time related constraints (Table 13, to 20) with Table 1, it is clear that the most flexible definitions exist in the approaches that are applicable in real scheduling situations.

Table 13 classifies approaches with capacity constraints. We distinguish models that impose set values (last column) and models with user definable capacity constraints (middle column). The latter obviously lead to

4 days	Warner and Prawda [134]: longer periods are not trustworthy with respect to personnel demand forecast
1 week	Aickelin and Dowsland [5], Aickelin and White [6], Dowsland [49]
2 weeks	Musa and Saxena [93]
	Blau and Sear [21]
3 weeks	Alfares [8]
2-4 weeks	Berrada et al. [19]
	Jan et al. [67]: maximum 30 days
2-6 weeks	Warner [132]
	Miller et al. [88]
1 month	Smith and Wiggins [124]
	Bellanti et al. [18]
	Ikegami and Niwa [64]
1 year	Chan and Weil [39]
User Defined	Burke et al. [27, 29, 32], De Causmaecker and Vanden Berghe [46]: usually 4 weeks

Table 11: Possible Planning Periods

User Definable Fixed Number	Ozkarahan and Bailey [106]
	Warner [132]: examples with 19-47 nurses
	Miller et al. [88]
	Musa and Saxena [93]: example with 11 people
	Aickelin and Dowsland [5], Aickelin and White [6], Dowsland [49]
	Chan and Weil [39]: 150 people
	Jan et al. [67]: maximum 30 people
	Bellanti et al. [18]
	Ikegami and Niwa [64]
	Burke et al. [27, 29, 32], De Causmaecker and Vanden Berghe [46]
To Be Minimised	Arthur and Ravindran [11]; the model relies on an even number of nurses to function properly
	Alfares [8]
	Easton and Mansour [51]

Table 12: Staff Size

more complex models. All the constraints restricting the amount of work that nurses carry out are considered: maximum number of assignments, overtime, maximum number of assignments per week or for particular shift types. A few models include all the listed constraints.

Table 14 groups approaches that allow nurses to set a few personal preferences. We do not include recurring preferences (such as preferably only weekend and night work) because we think that these should be arranged in the contracts. Some constraints are classified as 'general', if the paper does not specify what they mean. We have identified 4 specific types of personal preferences; requests for particular days on and off, and requests to work certain shifts on particular days.

Consecutiveness is a typical personnel rostering constraint. We have collected a large number of such constraints and grouped them in Tables 15 to 17. The first 3 constraints (Table 15) restrict the number of consecutive working days. Some flexibly allow the definition of a range between minimum and maximum but Aickelin and Dowsland [5] set a fixed number. Again, the right column of the table groups the least flexible approaches, in which the values are set by the developers. We also identified constraints on consecutive free days (bottom of Table 15 and top of Table 16). Models in which the users can freely set these constraints are rare (middle column). Patterns are cyclical time related constraints that are imposed on a non-cyclical schedule. Such constraints are dealt with by Warner [132], Burke et al. [27, 29, 31, 32], and De Causmaecker and Vanden Berghe [46]. The majority of developers apply set values (right column).

Capacity	User Definable	Set Values
Finite Capacity Constraints		
Maximum Number of Assignments	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]	Meyer auf'm Hofe [85]: 10 per 14 days, a legal constraint
	Bellanti et al. [18]	Miller et al. [88]: exact number of assignments: 10 per 14 days; hard constraint
		Kawanaka [71]: restrict free days to Saturdays, Sundays and bank holidays
Overtime	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]	Meyer auf'm Hofe [84]: minimise overtime
	Meisels et al. [80]	Chen and Yeung [41]
Maximum Number of assignments to a par ticular Shift Type	Burke et al. [27, 29, 31, 27], De Causmaecker and Vanden Berghe [46]	
	Ikegami and Niwa [64]	
Maximum Number of Shifts per Week	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]: can be set per shift type	
	Meisels et al. [80]	
	Berrada et al. [19]: hard constraint; restricted version limiting the assignments per week	

Table 13: Time related Constraints: Capacity

Personal Preferences	
General (unspecified)	Meyer auf'm Hofe [84]
	Warner [132]
Days Off	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46] with extra information about the type of absence (holiday, illness, educational)
	Miller et al. [88]: special requests can even overrule some of the feasibility constraints
	Chen and Yeung [41]
	Berrada et al. [19]
	Kawanaka [71]
	Okada [98], Okada and Okada [99]
	Meisels et al. [80]
	Ikegami and Niwa [64]
	Bellanti et al. [18]
Shifts Off	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]
Days On	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]: enable defining working days without specifying a particular shift type
	Berrada et al. [19] allow to ask for specific working days
Shift On	Burke et al. [29]
	Meisels et al. [80]
	Bellanti et al. [18]

Table 14: Time related Constraints: Personal Preferences

Consecutiveness	User Definable	Set Values
Consecutiveness Constraints (1)		
Maximum Number of Consecutive Days	Miller et al. [88]: non-binding constraint; non-binding maximum $<$ feasibility maximum	Arthur and Ravindran [11]: implicitly restricted to 12 because every other weekend is free
	Okada [98], Okada and Okada [99]	Chen and Yeung [41]: restricted to 6
	Berrada et al. [19]	Warner [132]
	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]	Miller et al. [88]: feasibility constraint: feasibility maximum
	Warner [132]: some workstretches allow for a 3 or 4 day weekend	Burns [36]: maximum 6
	Weil et al. [135]	
	Bellanti et al. [18]	
Minimum Number of Consecutive Days	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]	Weil et al. [135]: minimum 2; no isolated days on
	Miller et al. [88]: non-binding constraint: non-binding minimum $>$ feasibility minimum	Miller et al. [88]: feasibility constraint: feasibility minimum
	Bellanti et al. [18]	Warner [132]: no isolated working days
	Chen and Yeung [41]: minimum 2; on/off patterns are avoided	Berrada et al. [19]: minimum 2; no on/off patterns
	Warner [132]: off/on/off days are avoided	Jaszkiewicz [68] grouping working days
Number of Consecutive Days		Aickelin and Dowsland [5]: a set number, incorporated in the shift pattern cost
Maximum Number of Consecutive Free Days	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]	Miller et al. [88]: no patterns containing 4 consecutive days off
		Jaszkiewicz [68] grouping free days

Table 15: Time related Constraints: Consecutiveness Constraints (1)

There must be free time between consecutive assignments. Free time is user definable and it can differ per shift type (Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]). Meyer auf'm Hofe [85] applies a range (between obligatory and preferred) of spare time that is allowed.
Some consecutiveness constraints that arrange sequences of shift types are grouped in the last rows of Table 17. A very interesting type of constraint evaluates the balance of workload among personnel. Again, we distinguish more flexible models (middle column), and models that do not allow variance in the preset values (right column).

Weekend constraints cannot be ignored in personnel rostering. It is worth noting that the perception of well organised weekends (from anecdotal evidence) appears to be relatively more important for individual nurses than the satisfaction of any other type of time related constraint. Perhaps this is not so surprising. We have subdivided the constraints that relate to weekends into 4 groups in Table 19. The middle column presents the more flexible approaches and the last column groups the papers in which the constraints are determined beforehand by the developers.

Table 20 briefly describes constraints that did not match any of the constraint types described in Tables 13 to 19. We did not classify the constraint in the first row among the personal preference constraints of Table 14 because this one is not an occasional preference but rather one that determines contractual aspects (e.g. always free Wednesday afternoons). Jaszkiewicz [68] addresses the constraint that changes in shift types on consecutive days should be minimised. Other approaches apply a combination of consecutiveness constraints to reach that objective (see also Tables 15 to 17).
Finally, a few models incorporate a constraint on whether particular personnel members should work together or not. Such constraints do not appear in the more theoretical models.

Consecutiveness	User Definable	Set Values
Consecutiveness Constraints (2)		
Minimum Number of Consecutive Free Days	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]	Aickelin and Dowsland [5]: incorporated in the shift pattern cost
	Warner [132]: minimum 2; no isolated free days	Weil et al. [135]: minimum 2; no isolated free days
	Bellanti et al. [18]	Valouxis and Housos [130]
		Baker [13]: exactly 3 consecutive free days per week
		Chen and Yeung [41]: minimum 2; on/off patterns are avoided
		Miller et al. [88]: no on/off/on patterns, non-binding constraint
		Meyer auf'm Hofe [84]: preference of working time models (common sequences of working shifts), usually 2 weeks long
Patterns (these are cyclical time related constraints imposed on non-cyclical schedules)	Warner [132]: restricted number of 2 week patterns for day/night weeks and alternating free weekends; nurse specify their 'aversion' to certain patterns	Berrada et al. [19]: hard constraint; restricted constraint on weekend working patterns
	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]	Aickelin and Dowsland [5]: rotate night shifts and weekend work
		Miller et al. [88]: days on/off patterns only; less possible patterns than [132]
		Musa and Saxena [93]: nurses chose which one of 2 alternative weekends to be free
		Berrada et al. [19]: minimum 2, another aim is to group days off
		Arthur and Ravindran [11]: 5 possible shift patterns

Table 16: Time related Constraints: Consecutiveness Constraints (2)

Consecutiveness	User Definable	Set Values
Consecutiveness Constraints (3)		
Free Days after Night Shifts	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]: minimum number of free days after overnight shifts	Okada [98], Okada and Okada [99]: appropriate interval between series of night attendances
No night shift before day off	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]	
	Ikegami and Niwa [64]	
	Bellanti et al. [18]	
Time Between Assignments	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]: minimum rest time per shift	Kawanaka [71]: no early or day shift after a night shift
	Meyer auf'm Hofe [84, 85]: obligatory and preferred breaks between assignments	Warner [132]: not less than 16 hours between two assignments
Consecutive Shifts	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]: allow or forbid shift sequences on consecutive days	Kawanaka [71]: no early or day shift after a night
	Meyer auf'm Hofe [85]: minimum and preferred rest time between shifts	Bellanti et al. [18]: no morning shift after an afternoon
	Ikegami and Niwa [64]	
Sequences of Shift Types	Burke et al. [29]: flexible definition for each shift type	Meyer auf'm Hofe [85]: undesired sequences of work and free time
	Ikegami and Niwa [64]	
Mixture of Day-Night Shifts per Week		Aickelin and Dowsland [5]

Table 17: Time related Constraints: Consecutiveness Constraints (3)

Balance	User Definable	Set Values
Balance the Workload		
Balance	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]	Jaszkiewicz [68]: distribute the shifts evenly among nurses
	Meyer auf'm Hofe [85]: balance working time accounts	Okada [98], Okada and Okada [99]: balance night shifts, work on Sundays and work on bank holidays per month and per year
	Miller et al. [88]: individual 'aversion' coefficient for violations of non-binding constraints	Okada [98], Okada and Okada [99]: night shifts evenly distributed per day of the week (per person)
	Warner [132]: nurses and entire wards distribute a limited amount of penalty weights to constraints	Chiarandini et al. [44]: long-term fair distribution of undesired shifts as well as a distribution among personnel members of violations on the number of assignments per week
	Chen and Yeung [41]: some fairness measures	Aickelin and White [6]: evenly distribute unsatisfied requests and unpopular shifts
		Blau [20]
		Ikegami and Niwa [64]
		Bellanti et al. [18]: balance days off in weekends, balance shifts
		Kawanaka [71]: balance of shifts

Table 18: Time related Constraints: Balance the Workload

Weekends	User Definable	Set Values
Weekends		
Weekends in x Weeks	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]	
	Miller et al. [88]: per 4 or 6 weeks	
Complete Weekends and Extended Weekends	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]: weekends can be extended with Friday and/or Monday	Berrada et al. [19] encourage to extend the weekends
	Miller et al. [88]	
	Warner [132]: some work stretches allow for a 3 or 4 days weekend	
Compensation of Weekend Work	Meyer auf'm Hofe [85]	Okada [98], Okada and Okada [99]: days off to compensate for work on Sunday and half days off to compensate for 8-hours Saturday work; preferably within that week
Number of Consecutive Weekends	Warner [132] alternates free weekends (e.g. one every 2 or 3 weeks)	Arthur and Ravindran [11] strictly schedule every other weekend off
	Miller et al. [88]	Burns [36]: alternate weekends off
	Musa and Saxena [93]: alternate free and working weekends, individual nurses can choose which weekends are free	
	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]	

Table 19: Time related Constraints: Weekends

Other Constraints	User Definable	Set Values
Other Constraints		
Preference Days/Nights		Aickelin and Dowsland [5]
Working History	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]: influence on most constraint evaluations	Aickelin and Dowsland [5]: the cost of the previous period is added to the current one, with a maximum of 100
	Miller et al. [88]: historical schedule quality versus personal preferences	
	Ikegami and Niwa [64]	
Changes in Shifts on Consecutive Days		Jaszkiewicz [68]: changes should be minimised
Maximum Consecutive on/off/on Patterns		Miller et al. [88]: feasibility constraint
People Working Together or Not	Burke et al. [27, 29, 31, 32], De Causmaecker and Vanden Berghe [46]	Okada [98], Okada and Okada [99]: maximise the number of different personnel members for weekend and night work
	Jan et al. [67]: nurses have the right to select preferred partners	

Table 20: Time related Constraints: Others