Applied Soft Computing 12 (2012) 1180–1186

Contents lists available at SciVerse ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Firefly Algorithm for solving non-convex economic dispatch problems with valve loading effect

Xin-She Yang[a], Seyyed Soheil Sadat Hosseini[b], Amir Hossein Gandomi[c,*]

[a] Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, UK
[b] Young Researchers Club, Sari Branch, Islamic Azad University, Sari, Iran
[c] Young Researchers Club, Central Tehran Branch, Islamic Azad University, Tehran, Iran

ARTICLE INFO

Article history:
Received 5 August 2010
Received in revised form 21 June 2011
Accepted 22 September 2011
Available online 17 November 2011

Keywords:
Economic load dispatch
Valve loading effect
Firefly Algorithm
Metaheuristic

ABSTRACT

The growing costs of fuel and operation of power generating units warrant improvement of optimization methodologies for economic dispatch (ED) problems. The practical ED problems have non-convex objective functions with equality and inequality constraints that make it much harder to find the global optimum using any mathematical algorithms. Modern optimization algorithms are often meta-heuristic, and they are very promising in solving nonlinear programming problems. This paper presents a novel approach to determining the feasible optimal solution of the ED problems using the recently developed Firefly Algorithm (FA). Many nonlinear characteristics of power generators, and their operational constraints, such as generation limitations, prohibited operating zones, ramp rate limits, transmission loss, and nonlinear cost functions, were all contemplated for practical operation. To demonstrate the efficiency and applicability of the proposed method, we study four ED test systems having non-convex solution spaces and compared with some of the most recently published ED solution methods. The results of this study show that the proposed FA is able to find more economical loads than those determined by other methods. This algorithm is considered to be a promising alternative algorithm for solving the ED problems in practical power systems.

1. Introduction

Economic dispatch (ED) has become a fundamental function in operation and control of modern power systems. The ED problem can be stated as determining the least cost power generation schedule from a set of online generating units to satisfy the load demand at a given point of time [1]. Though the core objective of the problem is to minimize the operating cost satisfying the load demand, several types of physical and operational constraints make ED highly nonlinear constrained optimization problem, especially for larger systems [2]. However, accurate and intelligent scheduling of the units not only can decrease the operating cost significantly but also can assure higher reliability with improved security and less environmental impact [3]. In traditional ED approaches, the input–output characteristics (or cost function) of a generator is approximately shown by utilizing a single quadratic function. In practice, operating conditions of many generating units require the cost function to be modeled as a piecewise quadratic function [4]. However, higher-order nonlinearities and discontinuities are observed in real input–output characteristics, owing to valve-point loading in fossil fuel burning plants [5]. Besides, due to physical operation limitations (such as faults in the machines themselves) or the associated auxiliaries (such as boiler, feed pumps), units can have prohibited operating regions and generators that operate in these zones may experience amplification of vibrations in their shaft bearing, which should be prevented in practical applications [6]. Also due to the change restriction in the unit generation output, the units in the actual operation can have ramp rate limits [7]. So, ramp rate limits, prohibited operating zones (POZs) and valve loading effects should be considered to solve a realistic ED problem, which makes the finding of the optimum solution extremely hard.

Several deterministic optimization techniques were proposed to solve the ED problem, including gradient method [8], lambda iteration method [9], linear programming [10], quadratic programming [11], non-linear programming [12], Lagrangian relaxation algorithm [13] and dynamic programming [14]. By modeling of the final cost of generation accurately and taking valve-loading effect into account, the cost function of generators takes a non-convex form [15]. The theoretical assumptions behind previously mentioned algorithms (except dynamic programming) make them unsuitable for the ED formulation regarding non-convexity and differentiability. Furthermore, they are local optimizers by nature, i.e., they might converge to local solutions instead of global ones if the

* Corresponding author. Tel.: +1 234 788 0619.
E-mail addresses: xy227@cam.ac.uk (X.-S. Yang), s.sadathosseini@gmail.com (S.S. Sadat Hosseini), a.h.gandomi@gmail.com, ag72@uakron.edu (A.H. Gandomi).

1568-4946/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.asoc.2011.09.017

initial guess happens to be in the neighborhood of a local solution. Dynamic programming method may cause the dimensions of the ED problem to become extremely large, thus requiring enormous computational efforts.

To overcome these deficiencies, artificial intelligence methods have been used to solve the ED problem, and these methods include Genetic Algorithm (GA) [16], real-coded genetic algorithm (RCGA)[17], Tabu Search (TS) [5], Hopfield neural network [18], different types of Evolutionary Programming (EP) [19], biogeography-based optimization (BBO) [20], Evolutionary Strategy (ES) [21], Particle Swarm Optimization (PSO) ([22–25]), an improved coordinated aggregation-based particle swarm optimization (ICA-PSO) [26,27], Bacterial Foraging (BF) [28], harmony search (HS) [29] and Hopfield Neural Network (HNN) [30].

Although several optimization methodologies have been developed for the ED problem, the complexity of the task reveals the necessity for development of efficient algorithms to accurately locate the optimum solution. In this context, the objective of this work is to demonstrate a new approach for solving ED problems, aiming to provide a practical alternative for conventional methods. Here, Firefly Algorithm, developed by Yang [31], is used which has been successful to solve mixed variable and constrained engineering problems [32]. A more detailed description concerning theoretical and implementation feature of the proposed method is provided in the later sections. To show the efficiency and applicability of the proposed method, several types of ED problems are analyzed and results are compared with those available in the literature.

This paper is organized as follows. Section 2 illustrates the ED problem formulation considering valve-loading effect, prohibited operating zone (POZ) constraints and ramp rate limits. Moreover, the proposed method for constraints handling is demonstrated in this section. In Section 3, the Firefly Algorithm is described. In Section 4, simulation results are presented that demonstrate the potential of the proposed algorithm. Finally, Section 5 concludes the paper with discussions

2. Economic load dispatch problems with valve-point loadings problem

The goal of economic dispatch (ED) problem is to find the optimal combination of power generations that minimizes the total generation cost, while satisfying an equality constraint and inequality constraints. Cost efficiency is the most significant subproblem of power system operations. Owing to the highly nonlinearity characteristics of power systems and generators, ED belongs to a class of nonlinear programming optimization including equality and inequality constraints. Practically speaking, while the scheduled combined units for each specific period of operation are listed from unit commitment, the ED planning must carry out the optimal generation dispatch among the operating units in order to meet the load demands and practical operation constraints of generators, which consist of ramp rate limits, maximum and minimum limits, and prohibited operating zones. In general, the generation cost function is usually stated as a quadratic polynomial. Mathematically, the problem can be modeled as:

$$\min f = \sum_{i=1}^{n} F_i(P_i) \tag{1}$$

where F_i is the total generation cost for the generator unit i, which is defined by the following equation:

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i \tag{2}$$

where a_i, b_i and c_i are coefficients of generator i.

The valve-opening process of multivalve steam turbines produces a ripple-like effect in the heat rate curve of the generators. This curve contains higher order nonlinearity because of the valve-point effect, and should be refined by a sine function. Also the solution procedure can easily trap in the local minima in the vicinity of optimal value. To take account for the valve-point effects, sinusoidal terms are added to the quadratic cost functions as follows:

$$\tilde{F_i} = a_i P_i^2 + b_i P_i + c_i + \left| e_i \sin(f_i(P_i^{\min} - P_i) \right| \tag{3}$$

where e_i and f_i are constants of the unit with valve-point effects.

The model in Eq. (3) is subject to the following constraints:

2.1. Power balance constraint

$$\sum_{i=1}^{N_G} P_i = P_D + P_L \tag{4}$$

where P_D is the load demand and P_L is the total transmission network losses of system. To compute network losses, the B-coefficient method [7] is commonly utilized by the power utility industry. In the B-coefficient method, the transmission losses are expressed as a quadratic function:

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} B_{0i} P_i + B_{00}. \tag{5}$$

2.2. Ramp rate limits

One of the unrealistic assumptions that prevailed for simplifying the problem in many of the earlier research is that the adaptations to the power output are instantaneous. However, under practical circumstances, ramp rate limit restrains the operating range of all the online units for tuning the generator operation between two operating periods [33]. The generation may increase or decrease with corresponding upper and lower ramp rate limits. Therefore, units are restricted due to these ramp rate limits as mentioned below.

If power generation increases, we have

$$P_i - P_i^0 \le \mathrm{UR}_i. \tag{6}$$

If power generation decreases, we have

$$P_i^0 - P_i \le \mathrm{DR}_i \tag{7}$$

where P_i^0 is the previous power generation of unit i. UR_i and DR_i are the up-ramp and down-ramp limits of the ith generator, respectively. The inclusion of ramp rate limits changes the generator operation constraints (5) as follows:

$$\max(P_i^{\max}, \mathrm{UR}_i - P_{i)} \le P_i \le \min(P_i^{\max}, P_i^0 - \mathrm{DR}_i). \tag{8}$$

2.3. Prohibited operating zones

A generator with prohibited operating zones has discontinuous fuel-cost characteristics. The conception of prohibited operating zones is consisted of the following constraint in the ED:

$$\begin{cases} P_i^{\min} \le P_i \le P_{i,1}^{\mathrm{LB}} \\ P_{i,j-1}^{\mathrm{UB}} \le P_i \le P_{i,j}^{\mathrm{LB}} j = 2, 3, \ldots, \mathrm{NP}_i \\ P_{i,j}^{\mathrm{UB}} \le P_i \le P_i^{\max} j = \mathrm{NP}_i \end{cases}. \tag{9}$$

where $P_{i,j}^{\mathrm{LB}}$ and $P_{i,j}^{\mathrm{UB}}$ are the lower and upper boundaries of prohibited operating zone j of generator i, respectively; NP_i is the number of prohibited operating zones of generator i.

3. Firefly Algorithm

The Firefly Algorithm was developed by Yang ([34,35]), and it was based on the following idealized behavior of the flashing characteristics of fireflies:

- All fireflies are unisex so that one firefly is attracted to other fireflies regardless of their sex;
- attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less bright one will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If no one is brighter than a particular firefly, it moves randomly;
- the brightness or light intensity of a firefly is affected or determined by the landscape of the objective function to be optimized.

For a maximization problem, the brightness can simply be proportional to the objective function. Other forms of brightness can be defined in a similar way to the fitness function in genetic algorithms or the bacterial foraging algorithm (BFA) ([36]).

Firefly Algorithm
Objective function $f(x)$, $x = (x_1, \ldots, x_d)^T$
Initialize a population of fireflies $x_i (i = 1, 2, \ldots, n)$
Define light absorption coefficient γ
while (t < MaxGeneration)
for $i = 1$: n all n fireflies
  for $j = 1$: i all n fireflies
    Light intensity I_i at x_i is determined by $f(x_i)$
    if ($I_j > I_i$)
      Move firefly i towards j in all d dimensions
    end if
    Attractiveness varies with distance r via exp $[-\gamma r^2]$
    Evaluate new solutions and update light intensity
  end for j
end for i
Rank the fireflies and find the current best
end while
Postprocess results and visualization

The movement of a firefly i is attracted to another more attractive (brighter) firefly j is determined by

$$x_i^{t+1} = x_i^t + \beta_0 e^{-\gamma r_{ij}^2}(x_i^t - x_i^t) + \alpha \varepsilon_i^t \tag{12}$$

where β_0 is the attractiveness at $r = 0$, the second term is due to the attraction, while the third term is randomization with the vector of random variables ε_i being drawn from a Gaussian distribution. The distance between any two fireflies i and j at x_i and x_j can be the Cartesian distance $r_{ij} = \left\| x_i - x_j \right\|_2$ or the l_2-norm. For other applications such as scheduling, the distance can be time delay or any suitable forms, not necessarily the Cartesian distance. For most cases in our implementation, we can take $\beta_0 = 1, \alpha \in [0, 1]$, and $\gamma = 1$. In addition, if the scales vary significantly in different dimensions such as -10^5 to 10^5 in one dimension while, say, -10^{-3} to 10^3 along others, it is a good idea to replace α by αS_k where the scaling parameters $S_k (k = 1, \ldots, d)$ in the d dimensions should be determined by the actual scales of the problem of interest. In essence, the parameter γ characterizes the variation of the attractiveness, and partly controls how the algorithm behaves. It is also possible to adjust γ so that multiple optima can be found at the same during iterations.

3.1. Constraint handling

A significant factor in the application of optimization techniques is how the algorithm handles the constraints concerning the problem. The POZ constraints (9) are utilized as follows. If the generation of unit i is settled in its jth POZ, i.e.:

$$P_{i,j}^{\mathrm{LB}} \prec P_i \prec P_{i,j}^{\mathrm{UB}} \tag{13}$$

then the amount of generation is cut to the nearest boundary of the jth POZ as follows:

$$P_{i,j}^{\mathrm{ave}} = \left(\frac{P_{i,j}^{\mathrm{LB}} + P_{i,j}^{\mathrm{UB}}}{2} \right) \tag{14}$$

$$P_i = \begin{cases} P_{i,j}^{\mathrm{LB}} & \text{if } P_{i,j}^{\mathrm{LB}} \prec P_i \le P_{i,j}^{\mathrm{ave}} \\ P_{i,j}^{\mathrm{UB}} & \text{if } P_{i,j}^{\mathrm{ave}} \prec P_i \prec P_{i,j}^{\mathrm{UB}} \end{cases}. \tag{15}$$

For a nonlinear optimization problem with equality and inequality constraints, a common method is the penalty method. The idea is to define a penalty function so that the constrained problem is transformed into an unconstrained problem. Now we can define

$$\prod(x, \mu_i, \upsilon_j) = f(x) + \sum_{i=1}^{M} \mu_i \varphi_i^2(x) + \sum_{j=1}^{N} \upsilon_j \psi_j^2(x) \tag{16}$$

where $\mu_i \ge 1$ and $\upsilon_j \ge 0$ which should be large enough, depending on the solution quality needed. As we can see, when an equality constraint it met, its effect or contribution to $\prod$ is zero. However, when it is violated, it is penalized heavily as it increases $\prod$ significantly. Similarly, it is true when inequality constraints becomes tight or exactly. It should be mentioned that generation and ramp rate limits are similar type of constraints. These constraints together state the overall upper/lower generation limits of the units.

4. Implementation and numerical experiments

All the EAs for the ED problems are implemented using MATLAB™ 7.0 on a PC with a Pentium IV, Intel Dual core 2.2 GHz, 1 GB RAM. Owing to the random nature of the FA (and in fact all metaheuristic algorithms), their performance cannot be judged by the result of a single run. Many trials with independent population initializations should be made to obtain a useful conclusion of the performance of the approach. Therefore, the results should be analyzed using statistic measures such as mean and standard deviation. The best, worst and mean obtained in 100 trials are used to compare the performances of different EAs. To find the effectiveness of the proposed FA, the test results are also compared with the results already reported by the most recently published methods for solving the ED problem.

There are 4 important parameters in the Firefly Algorithm: α_0, β_0, γ, and the population size n. In order to obtain the right parameters, we have carried out a detailed parametric study by varying these parameters. More specifically, we varied α_0, β_0 from 0.1 to 1.0 with a step increase of 0.1, γ from 0.01 to 100 with a step increase of 0.01 up to 1, and then 5 up to 100. We also varied n from 5 to 100 with an interval of 5. By analyzing the optimal solutions for a wide range of test functions, we found that the best values or ranges of the parameters are: $\alpha_0 = 0.4$–0.9, $\beta_0 = 0.5$–1.0, $\gamma = 0.1$–10 and $n = 25$–50. For most problems, we can use the fixed value of $\beta_0 = 1$. Therefore, the parameters of the proposed FA to solve the ED problem in the four test cases are: $\beta_0 = 1$, $\gamma = 1/L$ (where L is the characteristic scale of the problem), $\alpha = 0.5$ and then is gradually reduced to 0.01 as iterations proceed. n varies from 25 to 50, depending on the complexity of the problem. In addition, the maximum number of function evaluations varies from 5000 to 50,000, depending on the size of the ED problem.

Table 1
Comparison of FA and global optimal on the three benchmark constrained problems.

No.	Decision var. no.		Constraint no.	Global optimal	Predicted by FA
	Continues	Discrete		$[X_{\min}; F_{\min}]$	$[X_{\min}; F_{\min}]$
1	1	1	1	[1.375, 1; 2.124]	[1.375, 1; 2.124]
2	2	1	2	[0.94194, 2.1, 1; 1.07654]	[0.94194, 2.1, 1; 1.07654]
3	3	4	8	[0.2, 1.280624, 1.954483, 1, 0, 0, 1; 3.557463]	[0.2, 1.280624, 1.954483, 1, 0, 0, 1; 3.557463]

Table 2
The best, average and worst results of different ED solution methods for the 3 unit test system.

Methods	Generation cost ($/h)				
	Best	Average	Worst	Standard deviation	No. of evaluation
GAB [19]	8234.08	NA	NA	NA	10,000
GAF [19]	8234.07	NA	NA	NA	10,000
CEP [19]	8234.07	8235.97	8241.83	NA	1000
FEP [19]	8234.07	8234.24	8241.78	NA	1000
MFEB [19]	8234.08	8234.71	8241.8	NA	1000
IFEP [19]	8234.07	8234.16	8234.54	NA	1000
FA	8234.07	8234.08	8241.23	3.63	5000

NA: not available.

4.1. Validation

Before solving economic dispatch problems, FA was benchmarked using three mixed-variable numerical examples which are given as follows in detail. The example 1 is chosen from Floudas [37] and the examples 2 and 3 are selected from Costa and Oliveria [38]. The results obtained by FA and the global optimal are presented in Table 1. As it can be seen from this table the FA obtained the global optimal solution correctly in all the three examples.

Example 1.

$$\text{Minimize}: f(x,y) = -y + 2x - \ln\left(\frac{x}{2}\right)$$

$$\text{Subject to}: g_1 = -x - \ln\left(\frac{x}{2}\right) + y \le 0$$

where $0.5 \le x \le 1.5$ and $y \in \{0, 1\}$.

Example 2.

$$\text{Minimize}: f(x,y) = -0.7y + 5(x_1 - 0.5)^2 + 0.8$$

$$\text{Subject to}: \begin{array}{l} g_1 = -\exp(x_1 - 0.2) - x_2 \le 0 \\ g_2 = x_2 + 1.1y + 1.0 \le 0 \\ g_3 = x_1 - 1.2y - 0.2 \le 0 \end{array}$$

where $0.2 \le x_1 \le 1.0$, $-2.22554 \le x_2 \le -1.0$ and $y \in \{0, 1\}$.

Example 3.

$$\text{Minimize}: f(x,y) = (y_1 - 1)^2 + (y_2 - 1)^2 + (y_3 - 1)^2 - \ln(y_4 + 1) + (x_1 - 1)^2 + (x_2 - 2)^2 + (x_3 - 3)^2$$

$$\text{Subject to}: \begin{array}{l} g_1 = y_1 + y_2 + y_3 + x_1 + x_2 + x_3 - 5.0 \le 0 \\ g_2 = y_3^2 + x_1^2 + x_2^2 + x_3^2 - 5.5 \le 0 \\ g_3 = y_1 + x_1 - 1.2 \le 0 \\ g_4 = y_2 + x_2 - 1.8 \le 0 \\ g_5 = y_3 + x_3 - 2.5 \le 0 \\ g_6 = y_4 + x_1 - 1.2 \le 0 \\ g_7 = y_2^2 + x_2^2 - 1.64 \le 0 \\ g_8 = y_3^2 + x_3^2 - 4.25 \le 0 \\ g_9 = y_2^2 + x_3^2 - 4.64 \le 0 \end{array}$$

where $0 \le x_1, x_2, x_3$ and $y_1, y_2, y_3, y_4 \in \{0, 1\}$.

Table 3
Output power of generators in the best result of the proposed FA for the 3 unit test system.

Unit	Power (MW)
1	300.267
2	149.733
3	400.000
Total generation (MW)	850
Generation cost ($/h)	8234.074

4.2. Economic dispatch problems

4.2.1. Case 1: 3 generating units

This test case includes three generating units. The expected load demand to be met by all the three generating units is 850 MW. The description of the system can be found from [19]. It has been reported in Ref. [5] that the global minimum found for the three-generator system is 8234.07. Based on the above mentioned parameters, the Firefly Algorithm has been executed for 100 trials with various starting points to verify its performance and efficiency. The best, average and worst of cost functions achieved by various methods are shown in Table 2. All methods give a similar ‘best solution’, whereas ‘average’ and ‘worst’ costs differ. Table 3 shows that the proposed method has accomplished in finding the global optimal solution presented in Ref. [5]. The average execution time of the FA for this test system is 2.07 s.

4.2.2. Case 2: 13 generating units

This test case consists of thirteen generating units; the complexity to the solution process has significantly increased. Inasmuch as this is a larger system with higher non-linearity, it has more local minima and thus it is difficult to attain the global solution. To be able to deal with more complicated case with highly non-linearity is one of the main goals of FA applications. The load demand of this test system is 1800 MW. Exactly the same data of all units as given in Ref. [19] will be utilized in this case. Table 4 shows the best, average and worst results of different ED solution methods among 100 trial runs in the same way as listed in Table 2. The outcomes of the other approaches shown in Table 4 have been directly quoted from their corresponding references (‘NA’ means the related result is not available in the corresponding reference). The average execution time of the FA for this test system is 10.36956 s. The computation times of the FA are not compared with the other ED solution

Table 4
The best, average and worst results of different ED solution methods for the 13 unit test system.

Methods	Generation cost ($/h)				
	Best	Average	Worst	Standard deviation	No. of evaluation
CEP [19]	18,048.21	18,190.32	18,404.04	NA	NA
PSO [2]	18,030.72	18,205.78	NA	NA	10,000
MFEP [19]	18,028.09	18,192	18,416.89	NA	NA
FEP [19]	18,018	18,200.79	18,453.82	NA	NA
IFEP [19]	17,994.07	18,127.06	18,267.42	NA	NA
EP–SQP [2]	17,991.03	18,106.93	NA	NA	10,000
HDE [39]	17,975.73	18,134.8	NA	NA	100
CGA_MU [40]	17,975.34	NA	NA	NA	NA
PSO–SQP [2]	17,969.93	18,029.99	NA	NA	10,000
HS [29]	17,965.62	17,986.563	18,070.176	26.3702	22,500
IGA_MU [40]	17,963.98	NA	NA	NA	NA
DEC(1)-SQP(1) [41]	17,963.94	17,973.13	17,984.81	NA	NA
ST-HDE [39]	17,963.89	18,046.38	NA	NA	100
FA	17,963.83	18,029.16	18,168.80	148.542	25,000

NA: not available.

methods, since the computation times of each ED method are computed on a different hardware. Output power of the generators of the 13 unit test system in the minimum solution of the FA is shown in Table 5.

4.2.3. Case 3: 40 generating units

The test system has forty generating units with non-convex fuel cost function incorporating valve loading effects. The required load demand to be met by all the forty generating units is 10,500 MW. The detailed information of generating units of test system is given in Ref. [19]. This case study has a larger and more complex solution space than all the previous case studies, and so any difference between different ED solution techniques can be better revealed in this test case. The Firefly Algorithm has been executed for a hundred times with various starting points. The obtained results of the proposed FA to resolve the ED problem for this test system are shown in Table 5. In this table, the detailed comparisons of the best, average and worst solutions of the proposed FA and most recently published ED solution methods are shown. As seen from Table 6, the best solution of the proposed method is better than those of all other methods, indicating FA's higher efficiency to solve the ED problem comparing with the other methods. Hence, for power system ED problems of greater size with higher non-linearities, the proposed method is proved to be the best approach among all the methods. The average execution time of the FA for this test system is 4.72801 s, and such a computation time to solve the ED problem is reasonable and practical. Detailed results of the optimal solution of the proposed method, including generation output of each unit for this test system, are shown in Table 7.

Table 5
Output power of generators in the best result of the proposed FA for the 13 unit test system.

Unit	Power (MW)
1	628.31852
2	149.59952
3	222.74912
4	109.86655
5	109.86655
6	109.86655
7	109.86655
8	60.00000
9	109.86655
10	40.00000
11	40.00000
12	55.00000
13	55.00009
Total generation (MW)	1800
Generation cost ($/h)	17963.83080

4.2.4. Case 4: 15 generating units

In this case study, all mentioned practical constraints and non-linear characteristics of the ED problem are included. The valve loading effects, ramp rate limits and POZs are considered for the units of this test system, whose data is given in Ref. [22]. The prohibited operating zones embedded in the 4 units, units 2, 5, 6, and 12. This problem proves particularly challenging because these zones result in a non-convex decision space where 192 convex subspaces can be constituted for the dispatch problem. The remaining units have simple operational zone. This challenging problem not only requires the proper implementation of the constraints, but also employs an efficient search in different subregions without wasting too much time on the prohibited regions. This means that number of fireflies should be sufficient enough so that they can distribute in most promising regions with high probability, while they can also fly or jump to different regions when necessary. Therefore, a fine balance between solution quality and computational effort is required. To ensure the global optimum solution is reachable, we have tried to do a few test runs by varying the number of fireflies and the total number of functional evaluations. From this initial learning experience, we can get a crude estimate what parameters are appropriate for this problem. We then use these as a basis for the more intensive 100 runs.

The comparison of the best, average and worst solutions of the proposed FA and most recently published ED solution methods, is shown in Table 8. Again, the FA offers an improved generation cost over the other methods, clearly showing the proposed approach of locating better solutions is superior to others. Detailed results of the optimal solution of the proposed method, discovered through the proposed method are shown in Table 9. All mentioned constraints were satisfied. The average execution time of the FA for this test system is 16.05 s, which is again a good computation time.

From our simulations, we observed that Firefly Algorithms have some advantages over other algorithms such as particle swarm optimization. By comparing with PSO and other algorithms, Firefly Algorithm has two superiorities: automatic subdivision and random reduction. The fireflies in the FA can automatically divide into subgroups so that these subgroups swarm around the multimodal optima. This makes it possible for the algorithm to find all global optima simultaneously. Thus, the algorithm is particularly suitable

Table 6
The best, average and worst results of different ED solution methods for the 40 unit test system.

Methods	Generation cost ($/h)				
	Best	Average	Worst	Standard deviation	No. of evaluation
HGPSO [42]	124,797.13	126,855.70	NA	1160.91	NA
SPSO [42]	124,350.40	126,074.40	NA	1153.11	NA
PSO [2]	123,930.45	124,154.49	NA	NA	10,000
CEP [19]	123,488.29	124,793.48	126,902.89	NA	NA
HGAPSO [42]	122,780.00	124,575.70	NA	906.04	NA
FEP [19]	122,679.71	124,119.37	127,245.59	NA	NA
MFEP [19]	122,647.57	123,489.74	124,356.47	NA	NA
IFEP [19]	122,624.35	123,382.00	125,740.63	NA	NA
TM [43]	122,477.78	123,078.21	124,693.81	NA	4050
EP–SQP [2]	122,323.97	122,379.63	NA	NA	10,000
MPSO [44]	122,252.26	NA	NA	NA	NA
ESO [21]	122,122.16	122,558.45	123,143.07	NA	75,000
HPSOM [42]	122,112.40	124,350.87	NA	978.75	NA
PSO–SQP [2]	122,094.67	122,245.25	NA	NA	10,000
PSO-LRS [23]	122,035.79	122,558.45	123,461.67	NA	20,000
Improved GA [45]	121,915.93	122,811.41	123,334.00	NA	100,000
HPSOWM [42]	121,915.30	122,844.40	NA	497.44	NA
IGAMU [16]	121,819.25	NA	NA	NA	NA
HDE [39]	121,813.26	122,705.66	NA	NA	100
DEC(2)-SQP(1) [46]	121,741.97	122,295.12	122,839.29	386.181	18,000
PSO [24]	121,735.47	122,513.91	123,467.40	NA	20,000
APSO(1) [24]	121,704.73	122,221.36	122,995.09	NA	20,000
ST-HDE [39]	121,698.51	122,304.30	NA	NA	100
NPSO-LRS [23]	121,664.43	122,209.31	122,981.59	NA	20,000
APSO(2) [24]	121,663.52	122,153.67	122,912.39	NA	20,000
SOHPSO [22]	121,501.14	121,853.57	122,446.30	NA	62,500
BBO [20]	121,479.50	121,512.06	121,688.66	NA	50,000
BF [28]	121,423.63	121,814.94	NA	124.876	10,000
GA–PS–SQP [47]	121,458.00	122,039.00	NA	NA	1000
PS [48]	121,415.14	122,332.65	125,486.29	NA	1000
FA	121,415.05	121,416.57	121,424.56	1.784	25,000

NA: not available.

Table 7
Output power of generators in the best result of the proposed FA for the 40 unit test system.

Unit	Power (MW)	Unit	Power (MW)
1	110.8099	21	523.2793
2	110.8059	22	523.2793
3	97.40230	23	523.2832
4	179.7332	24	523.2832
5	92.70700	25	523.2793
6	140.0000	26	523.2793
7	259.6004	27	10.0000
8	284.6004	28	10.0000
9	284.6004	29	10.0000
10	130.0028	30	87.8008
11	168.8008	31	189.9989
12	168.8008	32	189.9989
13	214.7606	33	189.9989
14	304.5204	34	164.8036
15	394.2801	35	164.8036
16	394.2801	36	164.8036
17	489.2801	37	110.0000
18	489.2801	38	110.0000
19	511.2817	39	110.0000
20	511.2817	40	511.2794
Total generation (MW)	10500		
Generation cost ($/h)	121415.0522		

for nonlinear, multimodal optimization problems. In addition, the randomness is reduced gradually by using a similar strategy as that used in simulated annealing, and this will speed up convergence when the global optimality is approaching.

Table 8
The best, average and worst result of different ED solution methods for the 15 unit test system including POZ constraints, ramp rate limits and transmission losses.

Methods	Generation cost ($/h)				
	Best	Average	Worst	Standard deviation	No. of evaluation
PSO [49]	32,858	33,039	33,331	NA	20,000
GA [49]	33,113	33,228	33,337	NA	20,000
SOH_PSO [22]	32,751	32,878	32,945	NA	62,500
CPSO1 [50]	32,835	33,021	33,318	NA	8000
CPSO2 [50]	32,834	33,021	33,318	NA	8000
BF [28]	32,784.5	32,796.8	NA	85.7743	10,000
FA	32,704.5	32,856.1	33,175.0	147.17022	50,000

NA: not available.

Table 9
Output power of generators and transmission losses in the best result of the proposed FA for the 15 unit test system.

Unit	Power (MW)
1	455.0000
2	380.0000
3	130.0000
4	130.0000
5	170.0000
6	460.0000
7	430.0000
8	71.7450
9	58.9164
10	160.0000
11	80.0000
12	80.0000
13	25.0000
14	15.0000
15	15.0000
Losses (MW)	30.6614
Generation cost ($/h)	32,704.4501

5. Conclusions

In this paper, we have presented a new approach to non-convex ED problems based on the FA. Many of the nonlinear characteristics of power systems such as valve-point loadings, ramp rate limits, and prohibited operating zones are considered for practical generator operation in the proposed method. Four test cases have been studied and comparisons of the quality of the solution and performance have been conducted against several of most recently published ED solution methods. Based on simulation results, the solution quality and reliability show the superiority of the FA over other approaches. The proposed algorithm capability and robustness make it suitable to solve complex optimization problems like non-convex ED. Future studies can focus on the inclusion of more realistic constraints to the problem structure. Large-scale, realistic ED problems can be attempted by the proposed methodology. In addition, it would be a good research topic to extend the proposed approach to solve mixed integer programming problems which are often NP-hard. A detailed parametric study of the algorithm may also prove fruitful.

References

[1] A.J. Wood, B.F. Wollenberg, Power Generation, Operation and Control, John Wiley and Sons, New York, 1996.
[2] T.A.A. Victoire, A.E. Jeyakumar, Hybrid PSO-SQP for economic dispatch with valve-point effect, Electr. Power Syst. Res. 71 (1) (2004) 51–59.
[3] M.A. Abido, Multiobjective evolutionary algorithms for electric power dispatch problem, IEEE Trans. Evol. Comput. 10 (3) (2006) 315–329.
[4] K.Y. Lee, A. Sode-Yome, J.H. Park, Adaptive hopfield neural networks for economic load dispatch, IEEE Trans. Power Syst. 13 (2) (1998) 519–526.
[5] W.-M. Lin, F.-S. Cheng, M.-T. Tsay, An improved tabu search for economic dispatch with multiple minima, IEEE Trans. Power Syst. 17 (1) (2002) 108–112.
[6] F.N. Lee, A.M. Breipohl, Reserve constrained economic dispatch with prohibited operating zones, IEEE Trans. Power Syst. 8 (1) (1993) 246–254.
[7] P.H. Chen, H.C. Chang, Large-scale economic dispatch by genetic algorithm, IEEE Trans. Power Syst. 10 (4) (1995) 1919–1926.
[8] J.C. Dodu, P. Martin, A. Merlin, J. Pouget, An optimal formulation and solution of short-range operating problems for a power system with flow constraints, Proc. IEEE 60 (1) (1972) 54–63.
[9] C.L. Chen, S.C. Wang, Branch-and bound scheduling for thermal generating units, IEEE Trans. Energy Convers. 8 (2) (1993) 184–189.
[10] J. Parikh, D. Chattopadhyay, A multi-area linear programming approach for analysis of economic operation of the Indian power system, IEEE Trans. Power Syst. 11 (February (1)) (1996) 52–58.
[11] J.I-Y. Fan, L. Zhang, Real-time economic dispatch with line flow and emission constraints using quadratic programming, IEEE Trans. Power Syst. 13 (May (2)) (1998) 320–325.
[12] J. Nanda, L. Hari, M.L. Kothari, Economic emission load dispatch with line flow constraints using a classical technique, IEE Proc. Gener. Transm. Distrib. 141 (1) (1994) 1–10.
[13] J.F. Bard, Short-term scheduling of thermal-electric generators using Lagrangian relaxation, Oper. Res. 36 (5) (1988) 756–766.
[14] P.G. Lowery, Generating unit commitment by dynamic programming, IEEE Trans. Power Apparat. Syst. PAS-85 (5) (1996) 422–426.
[15] I.G. Damousis, A.G. Bakirtzis, P.S. Dokopoulos, Network constrained economic dispatch using real-coded genetic algorithm, IEEE Trans. Power Syst. 18 (1) (2003) 198–205.
[16] C.L. Chiang, Genetic-based algorithm for power economic load dispatch, IEE Proc. Gener. Transm. Distrib. 1 (2) (2007) 261–269.
[17] S. Kumar, R. Naresh, Non-convex economic load dispatch using an efficient real-coded genetic algorithm, Appl. Soft Comput. 9 (1) (2009) 321–329.
[18] S. Ching-Tzong, L. Chien-Tung, New approach with a Hopfield modeling framework to economic dispatch, IEEE Trans. Power Syst. 15 (2) (2000) 541–545.
[19] N. Sinha, R. Chakrabati, P.K. Chattopadhyay, Evolutionary programming techniques for economic load dispatch, IEEE Trans. Evol. Comput. 7 (1) (2003) 83–94.
[20] A. Bhattacharya, P.K. Chattopadhyay, Solving complex economic load dispatch problems using biogeography-based optimization, Expert Syst. Appl. 37 (2010) 3605–3615.
[21] A. Pereira-Neto, C. Unsihuay, O.R. Saavedra, Efficient evolutionary strategy optimization procedure to solve the nonconvex economic dispatch problem with generator constraints, IEE Proc. Gener. Transm. Distrib. 152 (5) (2005) 653–660.
[22] K.T. Chaturvedi, M. Pandit, L. Srivastava, Self-organizing hierarchical particle swarm optimization for nonconvex economic dispatch, IEEE Trans. Power Syst. 23 (3) (2008) 1079–1087.
[23] A.I. Selvakumar, K. Thanushkodi, A new particle swarm optimization solution to nonconvex economic dispatch problems, IEEE Trans. Power Syst. 22 (1) (2007) 42–51.
[24] A.I. Selvakumar, K. Thanushkodi, Anti-predatory particle swarm optimization: solution to nonconvex economic dispatch problems, Electr. Power Syst. Res. 78 (2008) 2–10.
[25] K.T. Chaturvedi, M. Pandit, L. Srivastava, Particle swarm optimization with crazy particles for nonconvex economic dispatch, Appl. Soft Comput. 9 (2009) 962–969.
[26] J.G. Vlachogiannis, K.Y. Lee, Economic load dispatch—a comparative study on heuristic optimization techniques with an improved coordinated aggregation-based PSO, IEEE Trans. Power Syst. 24 (2) (2009) 991–1001.
[27] S.S. Sadat Hosseini, A.H. Gandomi, Discussion of economic load dispatch—a comparative study on heuristic optimization techniques with an improved coordinated aggregation-based PSO, IEEE Trans. Power Syst. 25 (1) (2010) 590.
[28] B.K. Panigrahi, V.R. Pandi, Bacterial foraging optimization: Nelder–Mead hybrid algorithm for economic load dispatch, IET Gener. Transm. Distrib. 2 (4) (2008) 556–565.
[29] L.S. Coelho, V.C. Mariani, An improved harmony search algorithm for power economic load dispatch, Energy Convers. Manage. 50 (2009) 2522–2526.
[30] H. Altun, T. Yalcinoz, Implementing soft computing techniques to solve economic dispatch problem in power systems, Expert Syst. Appl. 35 (2008) 1668–1678.
[31] X.S. Yang, Firefly algorithm, stochastic test functions and design optimization, Int. J. Bio-Inspired Comput. 2 (2) (2010) 78–84.
[32] A.H. Gandomi, X.S. Yang, A.H. Alavi, Mixed variable structural optimization using Firefly Algorithm, Comput. Struct. 89 (23–24) (2011) 2325–2336.
[33] H. MA, A.A. El-Keib, R.E. Smith, A genetic algorithm-based approach to economic dispatch of power systems, in: Proc. 1994 IEEE Conf., Miami, FL, April, 1994, pp. 212–216.
[34] X.S. Yang, Nature-Inspired Metaheuristic Algorithms, Luniver Press, UK, 2008.
[35] X.S. Yang, Firefly algorithms for multimodal optimization, in: O. Watanabe, T. Zeugmann (Eds.), Stochastic Algorithms: Foundations and Appplications, SAGA 2009, Lecture Notes in Computer Science, vol. 5792, Springer-Verlag, Berlin, 2009, pp. 169–178.
[36] K. Gazi, K.M. Passino, Stability analysis of social foraging swarms, IEEE Trans. Syst. Man Cybern. Part B – Cybern. 34 (2004) 539–555.
[37] C.A. Floudas, Nonlinear Mixed-Integer Optimization. Fundamentals and Applications, Oxford University Press, New York, USA, 1995.
[38] L. Costa, P. Oliveria, Evolutionary algorithms approach to the solution jof mixed integer non-linear programming problems, Comput. Chem. Eng. 21 (2001) 257–266.
[39] S.-K. Wang, J.-P. Chiou, C.-W. Liu, Nonsmooth/non-convex economic dispatch by a novel hybrid differential evolution algorithm, IET Gener. Transm. Distrib. 1 (5) (2007) 793–803.
[40] C.L. Chiang, Improved genetic algorithm for power economic dispatch of units with valve-point effects and multiple fuels, IEEE Trans. Power Syst. 20 (4) (2005) 1690–1699.
[41] L.D.S. Coelho, V.C. Mariani, Erratum correlation to combining of chaotic differential evolution and quadratic programming for economic dispatch optimization with valve point effect, IEEE Trans. Power Syst. 21 (3) (2006) 1465.
[42] S.H. Ling, H.H.C. Iu, K.Y. Chan, H.K. Lam, B.C.W. Yeung, F.H. Leung, Hybrid particle swarm optimization with wavelet mutation and its industrial applications, IEEE Trans. Syst. Man Cybern. Part B – Cybern. 38 (3) (2008) 743–763.
[43] D. Liu, Y. Cai, Taguchi method for solving the economic dispatch problem with nonsmooth cost functions, IET Gener. Transm. Distrib. 1 (5) (2007) 793–803.
[44] J.B. Park, K.S. Lee, J.R. Shin, K.Y. Lee, A particle swarm optimization for economic dispatch with nonsmooth cost functions, IEEE Trans. Power Syst. 20 (1) (2005) 34–42.
[45] S.H. Ling, F.H.F. Leung, An improved genetic algorithm with average-bound crossover and wavelet mutation operation, Soft Comput. 11 (1) (2007) 7–31.
[46] L.D.S. Coelho, V.C. Mariani, Combining of chaotic differential evolution and quadratic programming for economic dispatch optimization with valve-point effect, IEEE Trans. Power Syst. 21 (2) (2006) 989–996.
[47] J.S. Alsumait, J.K. Sykulski, A.K. Al-Othman, A hybrid GA–PS–SQP method to solve power system valve-point economic dispatch problems, Appl. Energy 87 (2010) 1773–1781.
[48] J.S. Al-Sumait, A.K. Al-Othmam, J.K. Sykulski, Application of pattern search method to power system valve-point economic load dispatch, Electr. Power Energy Syst. 29 (10) (2007) 720–730.
[49] Z.L. Gaing, Particle swarm optimization to solving the economic dispatch considering generator constraints, IEEE Trans. Power Syst. 18 (3) (2003) 1718–1727.
[50] C. Jiejin, M. Xiaqqian, L. Lixiang, P. Haipeng, Chaotic particle swarm optimization for economic dispatch considering the generator constraints, Energy Convers. Manage. 48 (2007) 645–653.