HEC MONTRÉAL | CANADA RESEARCH CHAIR IN DISTRIBUTION MANAGEMENT

Fifty Years of Vehicle Routing

by

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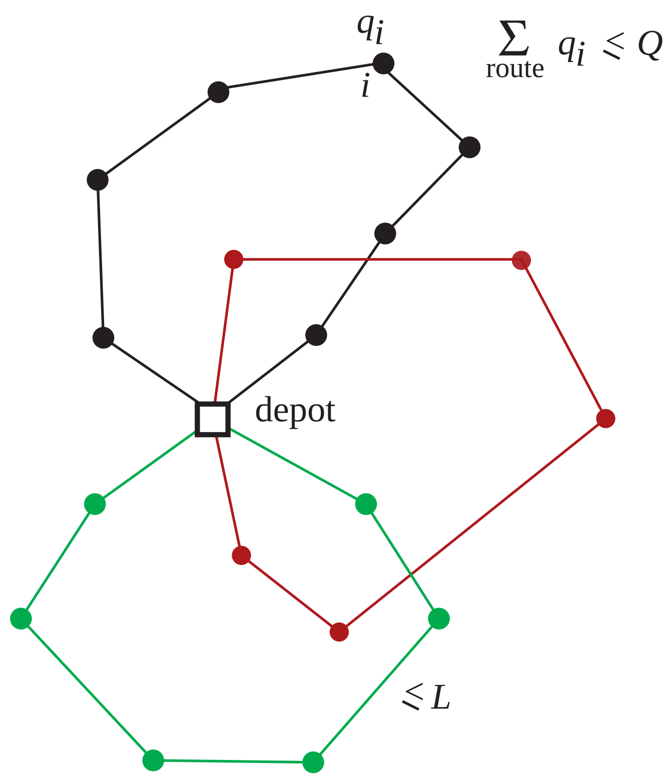
Vehicle Routing Problem

- Depot
- m (or at most m) identical vehicles based at the depot
- n customers
- Distance (cost, travel time) matrix (c_{ij})
- q_i: demand of customer i
- Q: vehicle capacity
- L: maximal route length (duration)

VRP: determine a set of m or at most m vehicle routes

1. Starting and ending at the depot
2. Visiting each customer exactly once
3. Satisfying the capacity constraint
4. Satisfying the maximal length constraint
5. Of minimal total cost



q_i
i
$\sum_{\text{route}} q_i \leq Q$
depot
$\leq L$

- NP-hard problem
- Has multiple applications
- Exact algorithms: relatively small instances
- In practice heuristics are used
- Several variants
  - heterogeneous vehicle fleet (Gendreau et al., 1999)
  - time windows (Cordeau et al., *VRP book*, 2002)
  - pickup and deliveries (Desaulniers et al., *VRP book*, 2002)
  - periodic visits (Cordeau et al., *Networks*, 1997), etc.
- Recommended books:
  - P. Toth and D. Vigo, *The Vehicle Routing Problem*, SIAM Monographs on Discrete Mathematics and Applications, Philadelphia, 2002.
  - B.L. Golden, S. Raghawan and E.A. Wasil, *The Vehicle Routing Problem*, Springer, New York, 2008.

Seminal Paper

- G.B. Dantzig and J.H. Ramser, "The Truck Dispatching Problem", *Management Science*, 6, 80–91, 1959.
- Heuristic
- Matching of vertices through (continuous) linear programming
- Respect of capacity constraints
- Introduction of "non-basic" pairings into solution through reduced cost criterion
- Elimination of fractional solutions through trial and error
- Example: one depot and seven customers

Exact Algorithms

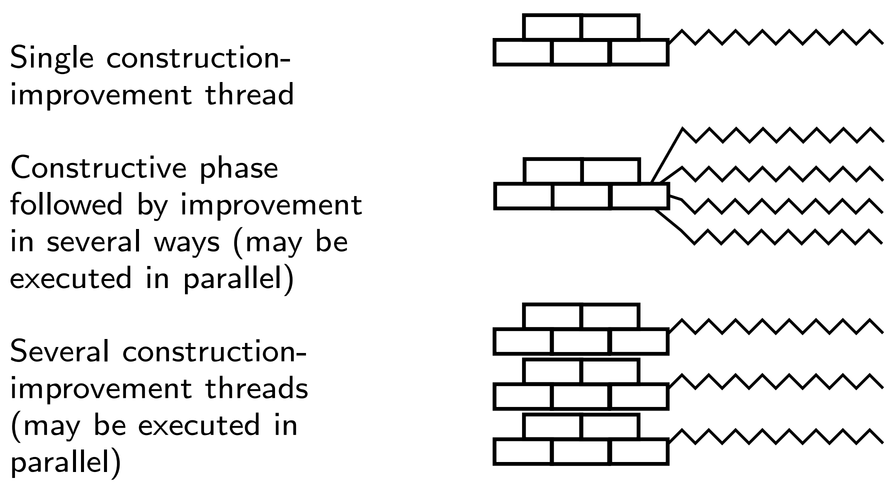
1981	Dynamic Programming with State Space Relaxation (Christofides, Mingozzi, Toth, *Networks*) $(10 \leq n \leq 25)$
	Branch-and-bound (k-shortest spanning trees, q-paths) (Christofides, Mingozzi, Toth, *Mathematical Programming* $(10 \leq n \leq 25)$
1985	Branch-and-cut (Laporte, Nobert, Desrochers, *Operations Research*) $(n \leq 60)$
1994	Branch-and-cut (for a restricted version of the VRP) (Fisher, *Operations Research*) $(n \leq 135)$
	Branch-and-cut (Ralphs et al., website) $(n \leq 101)$
	Branch-and-cut (Augerat et al., working paper) $(n \leq 135)$

2000	Branch-and-cut (Blasum and Hochstättler, working paper) $(n \leq 76)$
2002	Branch-and-cut (Naddef and Rinaldi, *VRP Book*) (survey)
2003	Branch-and-cut-and-price (Fukasawa et al., *Relatorios de Pesquisa en Engenharia de Produção*)
	Branch-and-cut (Wenger, Ph.D. dissertation, University of Heidelberg)
2004	Two-commodity network flow formulation (Baldacci, Hadjiconstantinou, Mingozzi, *Operations Research*) $(n \leq 135)$
2006	Branch-and-cut-and-price (Fukasawa et al., *Mathematical Programming*) $(n \leq 121)$
2008	Branch-and-cut-and-price (Baldacci, Christofides, Mingozzi) $(n \leq 121)$

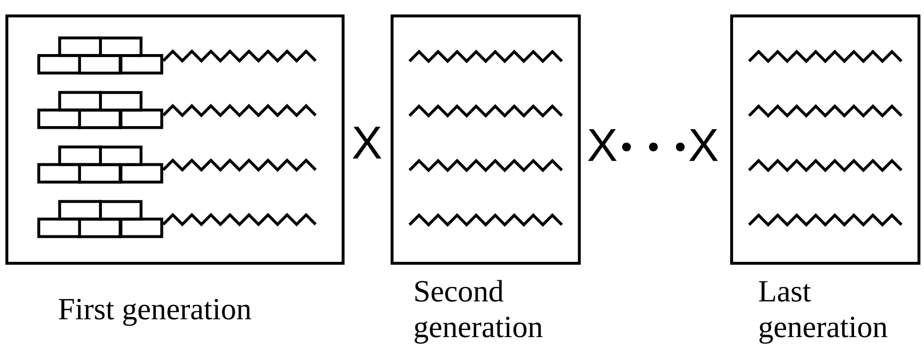
Heuristic Algorithms

- Classical algorithms (Laporte, Semet, *VRP Book*, 2002)
  - savings (Clarke, Wright, *Operations Research*, 1965)
  - sweep (Gillett, Miller, *Operations Research*, 1974)
  - cluster first, route second (Fisher, Jaikumas, *Networks*, 1981)
  - intra-route improvement methods (TSP heuristics)
  - inter-route improvement methods (λ-interchanges, Osman, 1993; cyclic exchanges, Thompson and Psaraffis, 1993; edge exchange schemes, Kindervater and Savelsbergh, 1997; ejection chains (Xu and Kelly, 1996; Rego and Roucairol, 1996; Rego, 1998); very large neighbourhood search (Ergun et al., 2003)
  - SERR (De Franceschi, Fischetti, Toth, working paper, 2004)

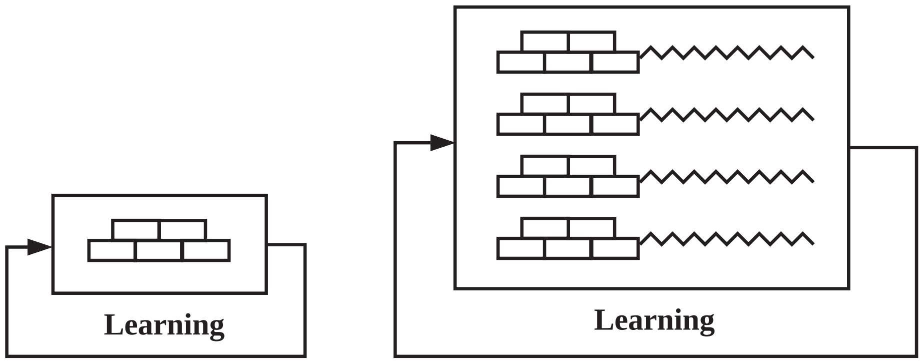
- Metaheuristics (Gendreau, Laporte, Potvin, *VRP Book*, 2002)
  - local search (simulated annealing, deterministic annealing, tabu search)



  - population search (adaptive memory procedures, genetic search)



– learning mechanisms (neural networks, ant colony systems)



a) Neural networks b) Ant algorithms

20 years of metaheuristics

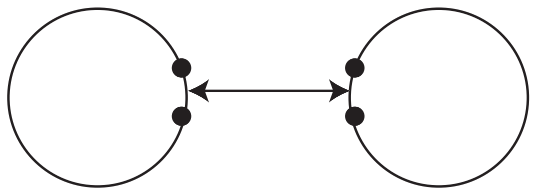
1989	First tabu search implementation (Willard, M.Sc. thesis, Imperial College)
1991	First version of Taburoute (Gendreau, Hertz, Laporte, Tristan I Conference)
1993	Tabu search (Taillard, *Networks*)
1993	Simulated Annealing and tabu search (Osman, *Annals of Operations Research*)
1994	Taburoute (Gendreau, Hertz, Laporte, *Management Science*)
1995	Adaptive memory (Rochat, Taillard, *Journal of Heuristics*)
1996	Ejection chains (Rego, Roucairol, *Meta-Heuristics: Theory and Applications*)

2001	Unified tabu search algorithm (Cordeau, Laporte, Mercier, *Journal of the Operational Research Society*)
2002	Adaptive memory (Tarantilis, Kiranoudis, *Annals of Operations Research*)
2003	Granular tabu search (Toth, Vigo, *INFORMS Journal on Computing*)
2003	Very large neighbourhood search (Ergun, Orlin, Steele-Feldman, working paper, MIT)
2004	Deterministic annealing (Li, Golden, Wasil, *Computers & Operations Research*)
2004	Population search (Prins, *Computers & Operations Research*; Mester and Bräysy, *Computers & Operations Research*)
2004	Ant systems optimization (Reinmann, Doerner, Hartl, *Computers & Operations Research*)
2005	Active guided evolution strategies (Mester, Bräysy, *Computers & Operations Research*

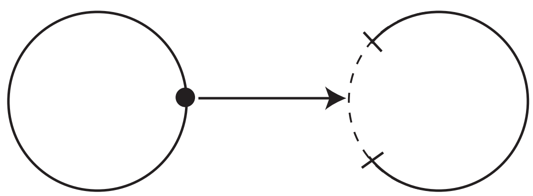
2005	Tabu search, adaptive memory
2006	Very large neighbourhood search (Ergun et al.)
2007	Attribute based hill climbing (Derigs, Kaiser)
2007	Genetic search + very large neighbourhood search (Mester and Bräysy)
2007	Guided very large neighbourhood search (Kytöjoki et al.)
2007	Adaptive very large neighbourhood search (Pisinger and Ropke)
2007	Memetic algorithm (Nagata)
2008	Local search limitation strategies (Nagata and Bräysy)
2009	Memetic algorithm (Nagata and Bräysy)
2009	GRASP + Evolutionary search (Prins)

Algorithmic ideas

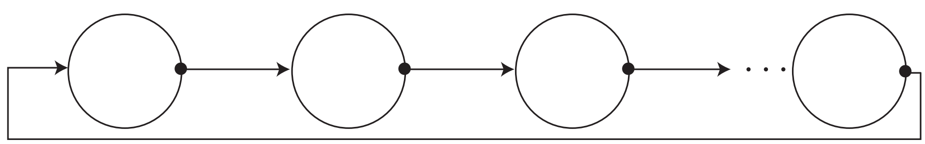
- Neighbourhood structures
  - 2-interchanges (Taillard, 1993)



  - simple vertex moves combined with local reoptimization (GENI) (Taburoute and UTSA)



  - composite moves (ejection chains, very large neighbourhood search) (Rego, Roucairol, 1995)



- Neighbourhood management
  - unique and simple neighbourhood structure
  - variable neighbourhood search (nested structure) (Mladenović, Hansen, 1997)
  - very large scale neighbourhood search (Ergun, 2001; Ergun, Orlin, Steele-Feldman, 2006)
  - destroy and repair (Shaw, 1997)
  - adaptive large scale neighbourhood search (Ropke and Pisinger, 2006)
  - limitation strategies in local search (Nagata, Bräysy, 2008)

- Tabu management
  - attribute sets
    $B(x) = \{(i,k) : i$ is visited by vehicle k
    in solution $x\}$
    Remove (i,k) from $B(x)$ and replace it
    with (i,k') $(k' \neq k)$
  - Assign tabu tag to an attribute (instead of maintaining an actual tabu list)
  - Tabu duration: variable in Taillard (1993) and in Taburoute, fixed but size-dependent in UTSA

- Aspiration criteria (overriding tabu status)
  - Attribute related in UTSA
- Intermediate infeasible solutions (Taburoute, UTSA)

  $$F'(x) = F(x) + \alpha Q(x) + \beta D(x)$$

  where α and β are periodically updated (almost necessary if simple vertex moves are used).

- Continuous diversification (Taillard)

  Penalize cost of worsening candidate solutions by adding to their cost a penalty proportional to the frequency of move:

  $$F(x) := F(x) + \gamma\sqrt{mn} f_{ik}$$

- Periodic route reoptimization
- False starts

  Used in Taburoute but not in UTSA: better perform 10^5 iterations on one solution than 10^4 iterations on each of 10 solutions.

- Intensification

  Used in Taburoute but not in UTSA.

- Data perturbation (Codenetti et al., *INFORMS Journal on Computing*, 1996)
  Used in Latest version of UTSA (0.69% $\rightarrow$ 0.56%): temporarily relocate the depot to next vertex of a route.

- Granularity (Toth, Vigo)
  - Remove long edges from data to obtain a sparse distance matrix.

    granularity threshold: $\nu = \beta\bar{c}$, where $\bar{c}$ is the average edge cost in a good feasible solution

    sparsification parameter $\beta \in [1.0, 2.0]$

    keep edges incident to the depot and those for which $c_{ij} \leq \nu$
  - Applied by Toth and Vigo: 4 times faster than Taburoute, also better.
  - Applied by Li, Golden, Wasil in conjunction with record-to-record principle (Dueck, 1993): accept candidate neighbour if cost does not exceed 1.01 times cost of best known solution.

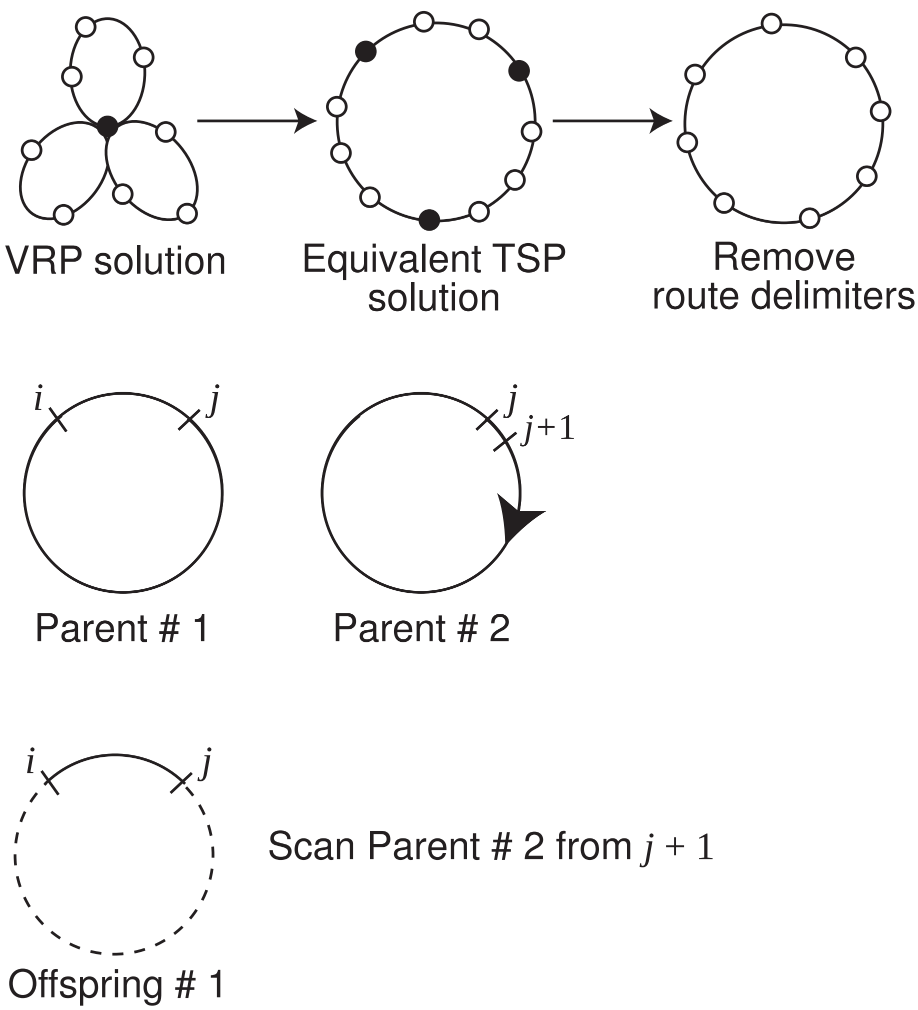
- Adaptive memory (Rochat, Taillard)

  Keep a pool of good solutions, combine them and reoptimize.

  - Rochat, Taillard: select a route from each of several solutions until this cannot be done without overlaps ($\rightarrow$ several routes + loose vertices). Reoptimize.
  - BoneRoute (Tarantilis, Kiranoudis): extract segments (bones) from good quality routes.

- Solution recombination (used in genetic search, Prins, 2004)

  Solution representation:



For offspring # 2 reverse the role of the two parents

- Guided evolution (AGES: active guided evolution strategy, Mester, Bräysy)

  Create each offspring from a single parent: apply local search, penalize some solution features (e.g. very long edges), use continuous diversification, 2-opt moves, 2-interchanges, very large neighbourhoods, restarts from best known solution.

- Memetic search (Moscatto and Cotta, 2003)

  Combines genetic search with local search. Improve offspring by local search.

  Advantage: provides width and depth. Applied by Prins, Mester and Bräysy, Nagata, Nagata and Bräysy.

- Learning (D-ants savings algorithm of Reimann, Doerner and Hartl)

  Generate a pool of good solution by Clarke and Wright savings algorithm and improve them. Replace saving criterion $s_{ij} = c_{i0} + c_{0j} - c_{ij}$ with $t_{ij}^{\alpha} s_{ij}^{\beta}$ where

  t_{ij}^{α} contains information on how good combining i and j turned out to be in previous solutions and α, β are user-controlled parameters.

  Apply saving s_{ij} with probability p_{ij}.

Christofides, Mingozzi, Toth (1979) instances $(51 \leq n \leq 199)$

Best ten heuristics

Authors (years)	**Heuristic**	**% above best**
Nagata (2007)	Memetic algorithm (best of 10)	0.00
Nagata, Bräysy (2008)	Local search limitation strategies (best of 10)	0.00
Nagata, Bräysy (2009)	Memetic algorithm (best of 10)	0.00
Rochat, Taillard (1995)	Tabu search, adaptive memory	0.00
Mester, Bräysy (2005)	Active guided evolution strategies (best)	0.03
Mester, Bräysy (2007)	Genetic search + very large neighbourhood search	0.03
Nagata (2007)	Memetic algorithm (average of 10)	0.03
Nagata, Bräysy (2009)	Memetic algorithm (average of 10)	0.03
Taillard (1993)	Tabu search	0.05
Nagata, Bräysy (2008)	Local search limitation strategies (average of 10)	0.05

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Golden, Wasil, Kelly, Chao (1998) instances $(200 \leq n \leq 480)$

Best ten heuristics

Authors (years)	Heuristic	% above best
Nagata, Bräysy (2008)	Local search limitation strategies (best of 10)	0.01
Nagata, Bräysy (2009)	Memetic algorithm (best of 10)	0.07
Nagata, Bräysy (2008)	Local search limitation strategies (average of 10)	0.13
Mester, Bräysy (2007)	Genetic search + very large neighbourhood search (best)	0.16
Mester, Bräysy (2005)	Active guided evolution strategies (best)	0.16
Nagata, Bräysy (2009)	Memetic algorithm (average of 10)	0.19
Prins (2009)	GRASP + evolutionary search	0.46
Pisinger, Ropke (2007)	Adaptive large scale neighbourhood search (best of 10)	0.65
Reimann, Doerner, Hartl (2004)	Ants	0.76
Tarantilis (2005)	Tabu search, adaptive memory (standard)	0.76

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Assessment

- Heuristics should be assessed on
  - accuracy
  - speed
  - simplicity
  - flexibility

(Cordeau, J.-F., Gendreau, M., Laporte, G., Potvin, J.-Y., Semet, F., "A guide to vehicle routing heuristics", *Journal of the Operational Research Society*, 53, 512–522, 2002)

- Recent heuristics are highly accurate.
- The best ones combine local search and population search (depth and breadth).
- Research should focus on simpler and more flexible algorithms (adaptable to VRP variants).