

# Metaheuristic Algorithms

- What is this course about?
- Difficult optimization problems
- Metaheuristics taxonomy
- Main topics, organization and evaluation rules

# What is this course about ?

- As almost all courses in Computer Science it is about **problem solving**
- Its main aim is to present techniques to solve **hard problems**
- There are problems which are hard:
  - both for humans and computers (e.g. large combinatorial optimization problems, multimodal / multiobjective optimization problems etc) – **computationally hard problems**
  - for computers but rather easy for humans (e.g. character recognition, face recognition, speech recognition etc) – **ill posed problem**

# Computationally hard problems

- Problems characterized by a large space of solutions for which there are no exact methods of polynomial complexity (so-called NP hard problems) – they are characterized by a search space of large size (which cannot be exhaustively searched)

## Examples:

- **Satisfiability problem (SAT)**: find the values of boolean variables for which a logical formula is true. For  $n$  variables the search space has the size  $2^n$
- **Travelling Salesman Problem (TSP)**: find a minimal cost tour which visits  $n$  towns. The search size is  $(n-1)!$  (in the symmetric case, it is  $(n-1)!/2$ )

# Ill-posed problems

- The particularity of problems which are easy for humans but hard for computers is that they are **ill-posed**, i.e. there is difficult to construct an abstract model which reflects all particularities of the problem
- Let us consider the following two problems:
  - Classify the employees of a company in two categories: first category will contain all of those who have an **income larger than the average salary per company** and the second category will contain the other employees
  - Classify the employees of a company in two categories: first category will contain all those which are **good candidates** for a bank loan and the second category will contain the other employees

# Ill-posed problems

- In the case of the first problem there is easy to construct a rule-based classifier:

IF income > average THEN Class 1  
ELSE Class 2

- In the case of the second problem it is not so easy to construct a classifier because there are a lot of other interrelated elements (health status, family, career evolution etc) to be taken into account in order to decide if a given employee is reliable for a bank loan. A bank expert relies on his **experience** (previous success and failure cases) when he makes a decision

# Well-posed vs Ill-posed Problems

- Differences between well-posed and ill-posed problems:

## Well-posed problems:

- There is an abstract model which describes the problem
- Consequently, there is a solving method, i.e. an algorithm

## Ill-posed problems:

- They cannot be easily formalized
- There are only some **examples** for which the results is known
- The data about the problem could be **incomplete** or **inconsistent**
- Thus, traditional methods cannot be applied

# Ill-posed problems

The methods appropriate for ill-posed problems should be characterized by:

- Ability to extract models from examples (**learning**)
- Ability to deal with dynamic environments (**adaptability**)
- Ability to deal with noisy, incomplete or inconsistent data (**robustness**)
- Ability to provide the answer in a reasonable amount of time (**efficiency**)

Designing a system having all these characteristics usually leads to solving an **optimization** problem (= find the **design variables** which **minimize** an error, minimize a cost or **maximize** a quality criteria)

# Optimization problems

**Rough description:** find one or several elements in the search space which optimize (minimize or maximize) one or several optimization criteria (called objective functions) and satisfy one or several constraints

**Mathematical description (single objective optimization):**

Find  $x^* \in D \subset R^n$  such that

$f(x^*) \leq f(x)$  for any  $x \in D$  ( $f : D \rightarrow R$  is the objective function)

$g_i(x^*) = 0, i = \overline{1, p}$  (equality constraints)

$h_j(x^*) \geq 0, j = \overline{1, q}$  (inequality constraints)



# Optimization challenges

Search space = space where the design variables take values

- Large size (many design variables)
- Characterized by complex constraints (the feasible region is not easy to reach)

Remark:

- Traditional techniques (e.g. mathematical programming) are either inapplicable or inefficient

Objective function

- „black-box” function (only the values of the function can be computed; no knowledge of its properties)
- many optima (multi-modal optimization)
- noisy function: different evaluations for the same argument may lead to different values (noisy or dynamic optimization)
- Multiple (conflictual) objective criteria (multi-objective optimization)

# Classes of optimization problems

- Discrete search space → combinatorial optimization

## Examples:

- Routing (e.g. vehicle routing)
- Planning (e.g. timetabling, task scheduling)
- Allocation (e.g. resource allocation)
- Selection (e.g. feature selection)

- Continuous search space → continuous optimization

## Examples:

- Parameter estimation
- Finding minimal energy configurations
- Training adaptive systems

# Routing

## Vehicle Routing Problem:

- Find a minimal cost route which visit a set of locations and satisfy some constraints

**Particular case:** travelling salesman problem

TSP= find a hamiltonian circuit of a minimal cost in a complete graph

## Solution representation:

### 1. Binary allocation matrix (nxn):

$A_{ij} = 1$  if node  $j$  is visited at step  $i$   
 $= 0$  otherwise

### Constraints:

- Each row contains exactly one 1
- Each column contains exactly one 1

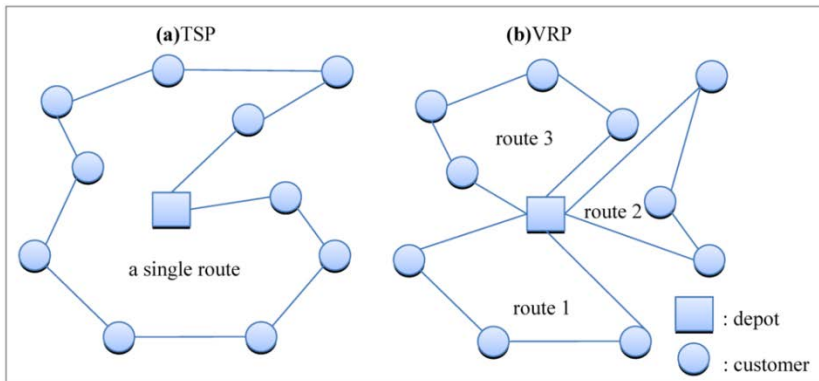
**Search space:** set of binary matrices of size  $n \times n$

**Search space size:**  $2^{n \times n}$

**Objective function (to be minimized):**

$$f(A) = \sum_{i=1}^n \sum_{j \neq k} c_{jk} A_{ij} A_{i+1,k}, \quad (n+1 \equiv 1)$$

$c_{jk}$  = cost of visiting the edge  $(j,k)$



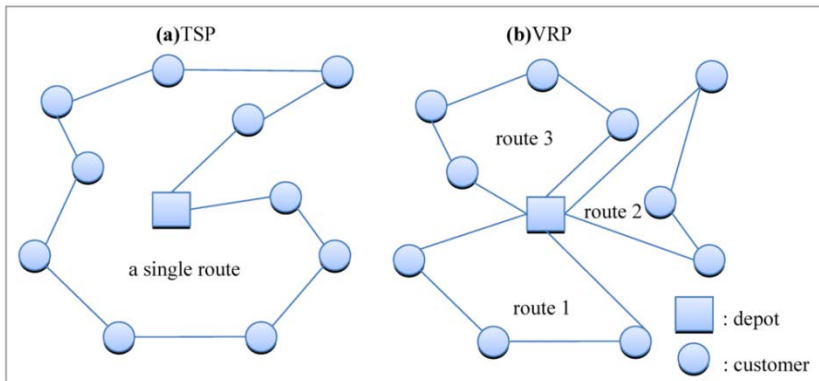
# Routing

## Vehicle Routing Problem:

- Find a minimal cost route which visit a set of locations and satisfy some constraints

Particular case: travelling salesman problem

TSP= find a hamiltonian circuit of a minimal cost in a complete graph



## Solution representation:

### 2. Permutation:

$p_i$  = index of node which is visited at step  $i$

### Constraints:

- The elements of  $p$  are distinct

Search space: the set of permutations of order  $n$  (order  $n-1$  for symmetric problems)

Search space size:  $n!$  (for symmetric problems  $(n-1)!/2$ )

Objective function (to be minimized):

$$f(p) = \sum_{i=1}^n c_{p_i p_{i+1}}, \quad (n+1 \equiv 1)$$

# Planning

**Problem:** Let us consider

- a set of events (e.g. lectures, exams etc.),
- a set of rooms
- a set of time slots

Construct a schedule such that each event is assigned to a room and a time slot such that different types of constraints are satisfied.

The constraints can be:

- hard (mandatory)
- soft (can be violated)

	rooms		
	R1	R2	R3
Time slots	T1	E1	E9
	T2	E4	E8
	T3	E6	E5
	T4	E2	E7

**Search space:** the set of functions which associate to each pair (time slot, room) an event (or the empty activity)

**Search space size (no constraints):**  $(k+1)^{mn}$

k events

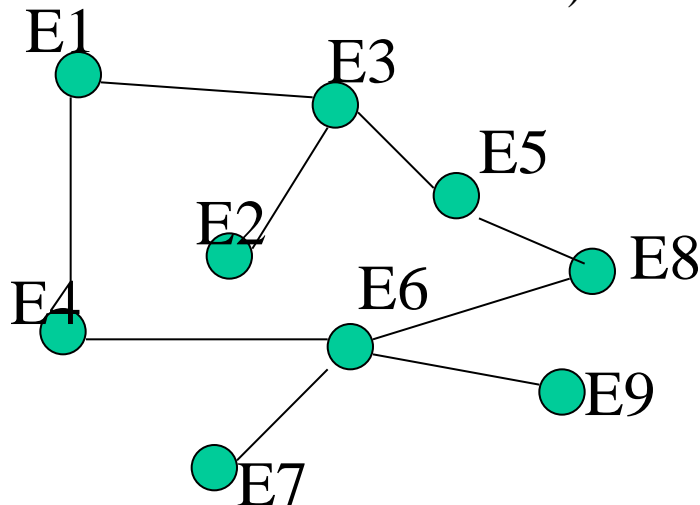
m time slots

n rooms

# Planning

- Hard constraints (the solution is feasible only if they are satisfied):
  - Each event is scheduled exactly once
  - Only one event is scheduled in a room at a given time moment
  - The room satisfies the requirements of the event
  - There are simultaneous events at which same persons should attend

Conflict graph (two connected nodes correspond to events which cannot be simultaneous)



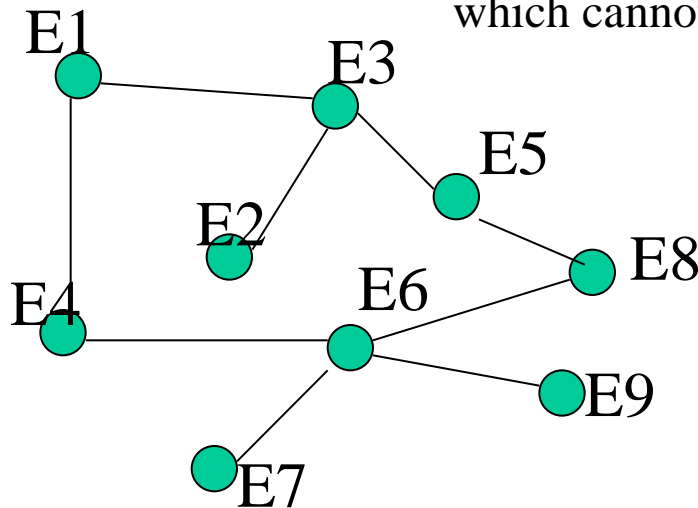
	R1	R2	R3
T1	E1	E3	E9
T2	E4		E8
T3	E6	E5	
T4	E2		E7

# Planning

- Soft constraints:
  - There are no more than k successive events which should be attended by the same participant
  - There are no participants for which there is only one event/day

**Idea:** the soft constraints can be transformed in optimization criteria (e.g. The number of participants for which some of the constraints are violated is as small as possible)

Conflict graph (two connected nodes correspond to events which cannot be simultaneous)



	R1	R2	R3
T1	E1	E3	E9
T2	E4		E8
T3	E6	E5	
T4	E2		E7

# Resource allocation

**Problem:** cloud resource allocation

Let us consider:

- A set of tasks characterized by some requirements
- A set of virtual machines (VMs) having some characteristics

The aim is to assign the tasks to VMs such that:

- The tasks requirements are satisfied
- The number of used virtual machines (or the global cost or the consumed energy) is as small as possible

**Particular case:** bin packing

**Solution representation:**

(n tasks, m VMs)

Allocation:  $V=(v_1, v_2, \dots, v_n)$

$v_i$  = index of the VM on which is placed the task  $i$

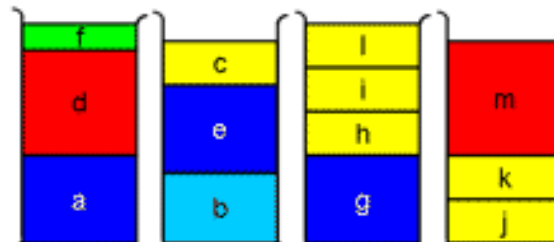
**Search space:** the set of functions defined on  $\{1, 2, \dots, n\}$  and taking values in  $\{1, 2, \dots, m\}$

**Search space size:**  $m^n$

**Objective function:**

$$f(V) = \sum_{i=1}^n \text{cost}(i, V_i)$$

Cost for assignment of task  $i$  on  $v_i$





# Selection

## Feature/attribute selection

Let us consider:

- a dataset characterized by a large number of attributes

We are looking for a subset of features:

- Which maximizes the classification accuracy
- Minimizes the cost of data processing (and/or the size of the model extracted from data)

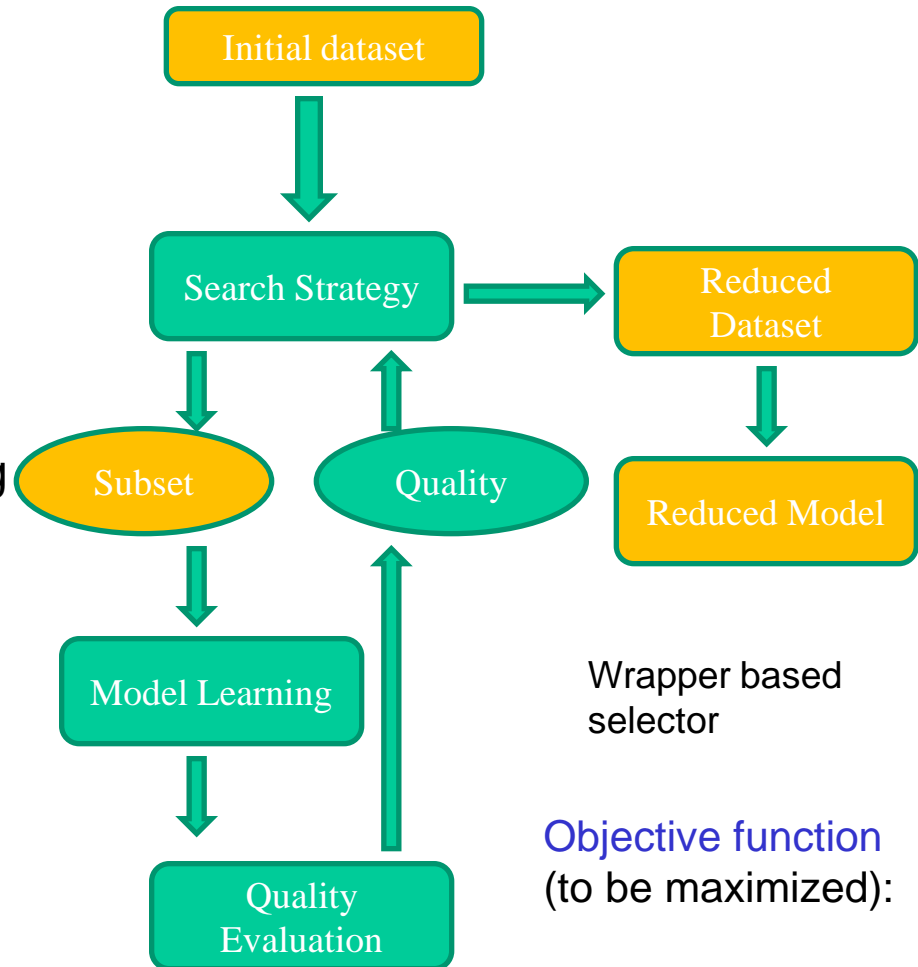
**Solution representation:** binary vector

$S_i=1$  if attribute  $i$  is selected

$=0$  if attribute  $i$  is not selected

**Search space:** the set of binary vectors with  $n$  elements

**Search space size:**  $2^n$  ( $n$ =initial number of attributes)



**Objective function**  
(to be maximized):

accuracy of the  
classification model

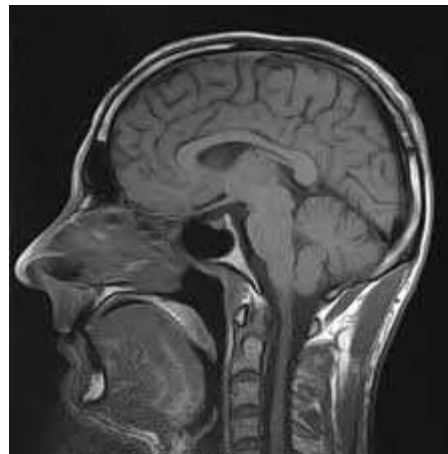
# Parameter estimation

## Image registration

Let us consider two images of the same object which should be „aligned” = we search for an image transformation which maximizes the similarities between the corresponding pixels of transformed images



$I_1$



$I_2$

## Solution representation:

Vector of real values corresponding to the parameters of the transformation (e.g. rotation, translation etc.)

## Search space:

$[a_1, b_1] \times [a_2, b_2] \times \dots \times [a_n, b_n]$

Objective function (to be minimized wrt the set of parameters,  $p$ )

$\text{Dist}(I_1, T(p; I_2))$  = distance between a reference image ( $I_1$ ) and the transformed image (using the set of parameters  $p$ )

# Parameter estimation

**Problem:** Find the geometric coordinates of a set of atoms which minimizes the internal energy of the system (there are different forms of the internal energy which are determined by the used potential function, e.g. Lennard-Jones)

**Solution representation:** vector of real values containing triplets of atoms coordinates

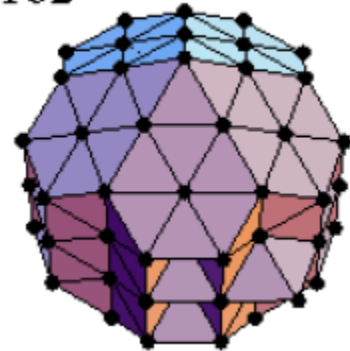
**Search space:**  $[a_1, b_1] \times [a_2, b_2] \times \dots \times [a_n, b_n]$   
(for instance:  $[0, 4] \times [0, 4] \times [-4, 4] \times \dots \times [-4, 4]$ )

**Challenge:** it is difficult to find the global optimum (particularly for large values of the number of atoms)

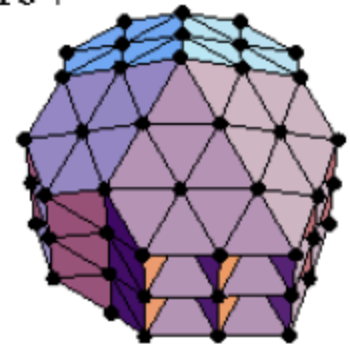
$$f(P_1, P_2, \dots, P_{3n}) = \sum_{i=1}^{n-1} \sum_{j=i+1}^n \left( \frac{\sigma^{12}}{\text{dist}(p_i, p_j)^{12}} - \frac{\sigma^6}{\text{dist}(p_i, p_j)^6} \right),$$

$$p_i = (P_{3(i-1)+1}, P_{3(i-1)+2}, P_{3(i-1)+3}) \quad \text{Metaheuristics - Lecture 1}$$

$N = 102$

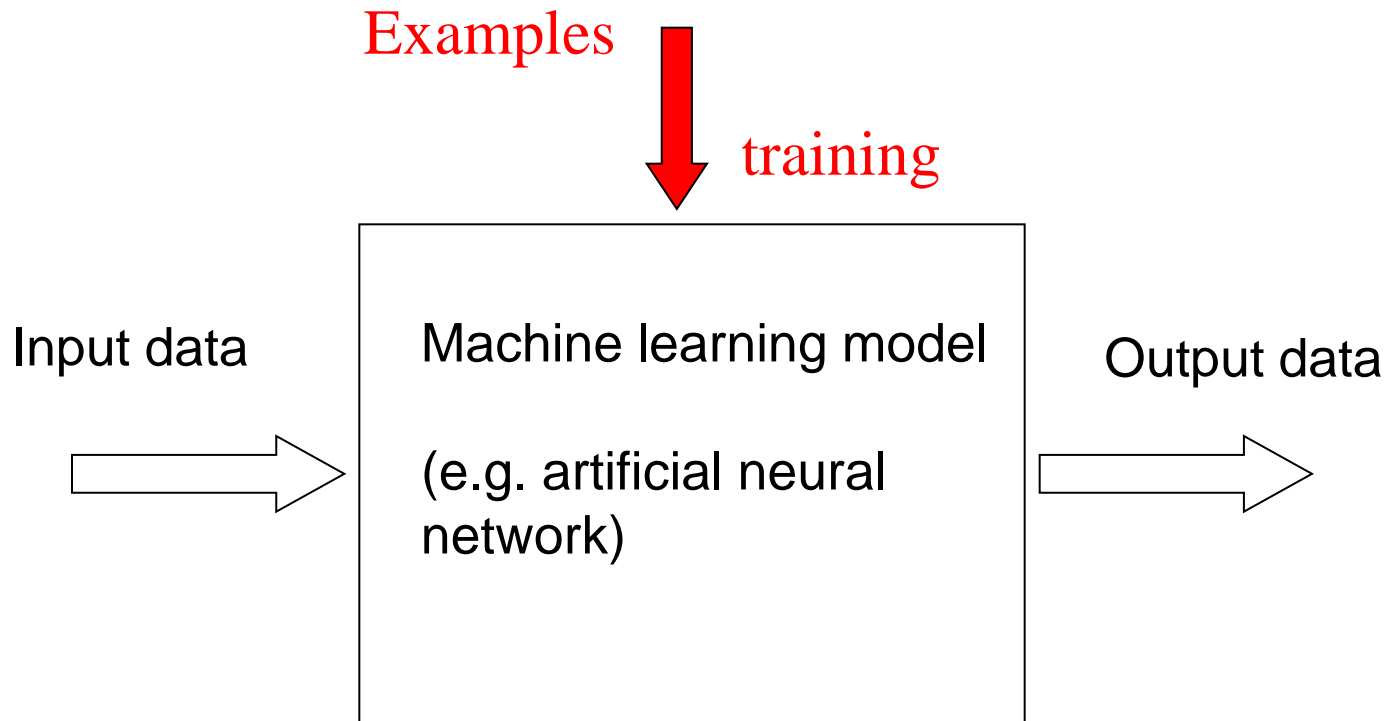


$N = 104$



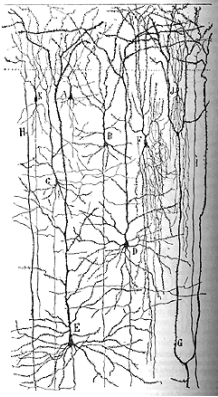
# Training adaptive systems

Machine learning approach



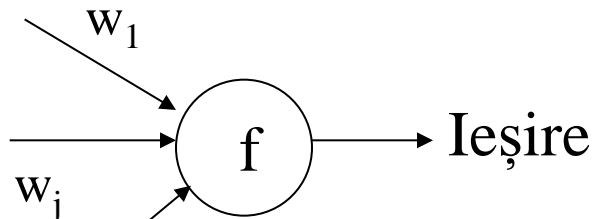
**Training** = estimate the adaptive parameters of the model such that an error function is minimized or a quality criterion is maximized

# Artificial neural networks



ANN= set of interconnected artificial neurons (functional units characterized by adaptive parameters – the weights)

inputs



Weights of connections

$$y = f\left(\sum_{j=1}^n w_j x_j - w_0\right)$$

$$u = \sum_{j=1}^n w_j x_j - w_0$$

Aggregation function

$$f(u) = \text{sgn}(u)$$

$$f(u) = \tanh(u)$$

$$f(u) = H(u)$$

$$f(u) = \frac{1}{1 + \exp(-u)}$$

Activation function

(computation of the output signal)

# ANN training

Structure of an artificial neural network:

- Architecture
- Functioning
- **Supervised training:** find the weights which minimizes an error function (difference the expected answer and that produced by the network)

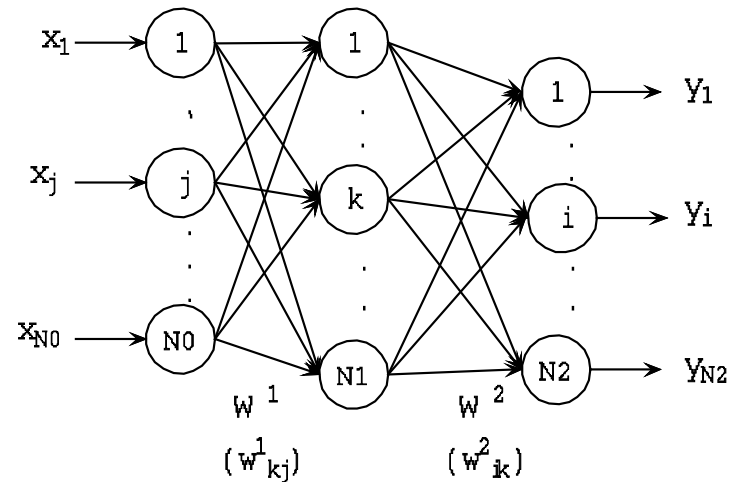
**Search space:** the space of the weights  $W$  (vectors with  $(N_0+1) \times (N_1+1)$  components)

**Objective function:**

$$f(W) = \sum_{l=1}^L \sum_{i=1}^{N_2} (d_i^l - y_i^l(W))^2$$

$d_1^l, \dots, d_{N_2}^l$  – correct answer for example  $l$

$y_1^l(W), \dots, y_{N_2}^l(W)$  – network answer for example  $l$

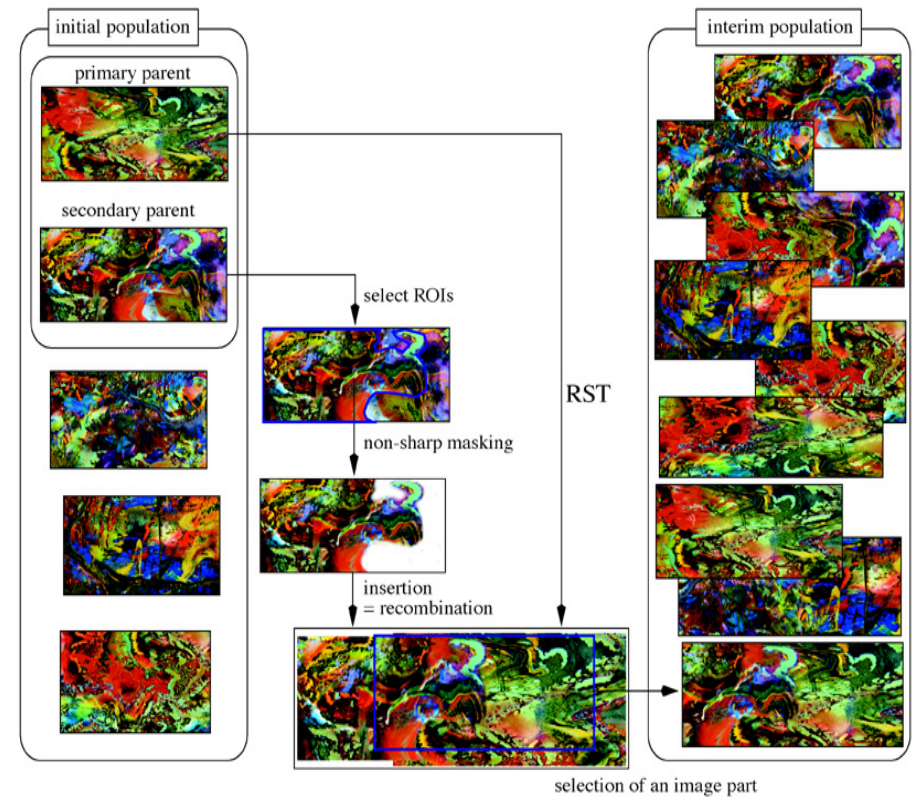


$$y_i = f \left( \sum_{k=0}^{N_1} w_{ik}^2 f \left( \sum_{j=0}^{N_0} w_{kj}^1 x_j \right) \right), \quad i = \overline{1, N_2}$$

Feedforward neural network  
(used for classification/association problems)

# Interactive optimization

- Design of various artistic structures (images, sounds) which optimizes a subjective criterion provided by a user
- The objective function is evaluated by the user (e.g. a score on artistic quality of the structure)
- DarwinTunes  
<http://darwintunes.org/>



[<http://www.evogenio.com/de/GBEvoArt/EvoArt1.html>]

# Metaheuristic Algorithms

“A metaheuristic is formally defined as an **iterative generation process** which guides a **subordinate heuristic** by **combining intelligently different concepts** for **exploring** and **exploiting** the search space, **learning strategies** are used to structure information in order to find efficiently **near-optimal** solutions”

[I.H. Osman, G. Laporte, Metaheuristics: a bibliography, Annals of Operations Research 63 (1996) 513–623]

“A **top-level general strategy** which **guides other heuristics** to search for feasible solutions in domains where the task is hard.”

[dictionary.reference.com]

## Terminology

- **Heuristic** (from greek: heuriskein = to find, to discover)
- **Meta** (from greek: beyond in the sense of high-level)



# Metaheuristic Algorithms

## Key ideas:

- Metaheuristics rely on iterative improvements of candidate solutions by using in a clever way heuristics which are specific for the problem to be solved
- The main aim is to ensure a **balance** between
  - **exploration** of the search space
  - **exploitation** of the information gathered during the previous search
- They are rather general techniques (**same algorithm** can be applied with minor changes to **different problems**)
- Most of them rely on the usage of stochastic operators (involve **randomness** to compensate for the lack of knowledge on the problem properties)

# Metaheuristic Algorithms

## Advantages:

- They require only the values of the objective function (which does not have to be smooth or continuous, i.e. **black box optimization**)
- They can estimate the global optimum (if the search space is properly explored)

## Disadvantages:

- There are **few theoretical results** concerning the convergence and the quality of the approximation (they are not as sound as mathematical programming methods)
- Their design is mainly based on **intuition** and **analogy** with other processes (in many cases observed in nature or in social interactions) which ensure the improvement in some sense of the behavior of a system

# Metaheuristics Taxonomy

W.r.t. search scheme:

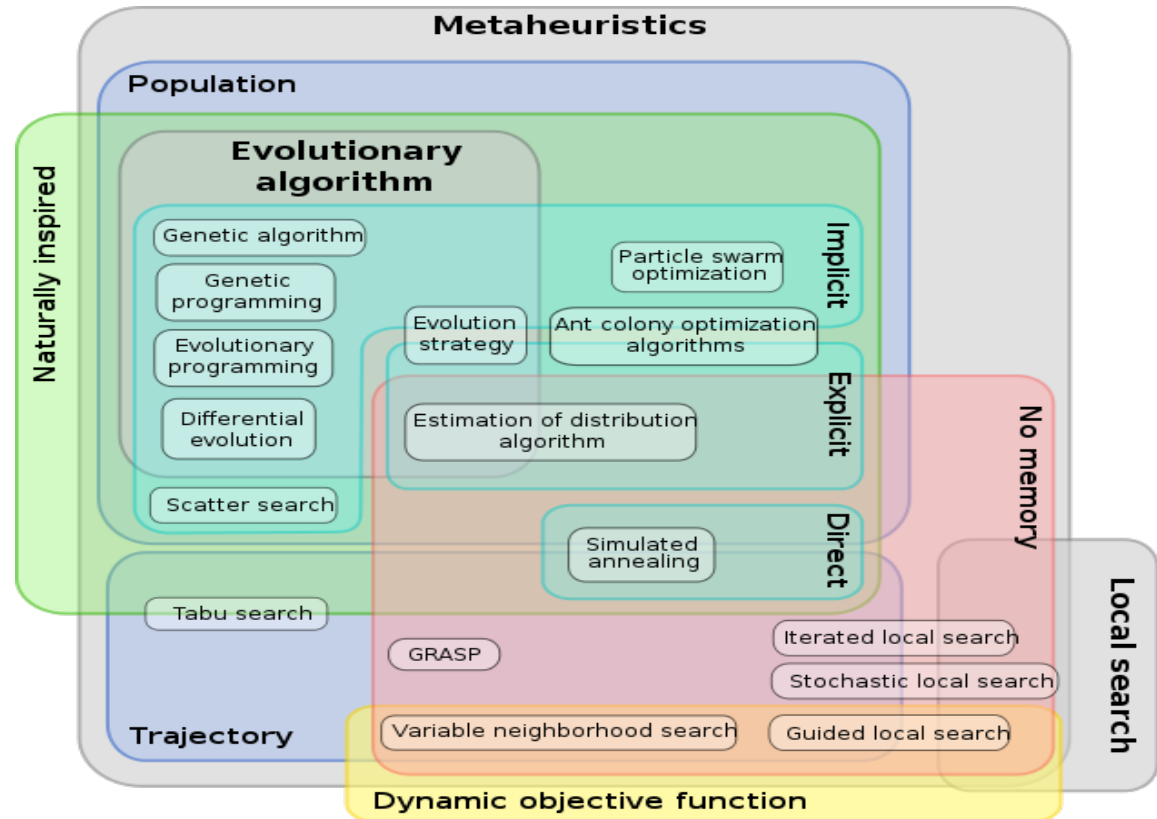
- Trajectory based (one candidate solution)
- Population based (several candidate solutions)

W.r.t. search area:

- Local
- Global

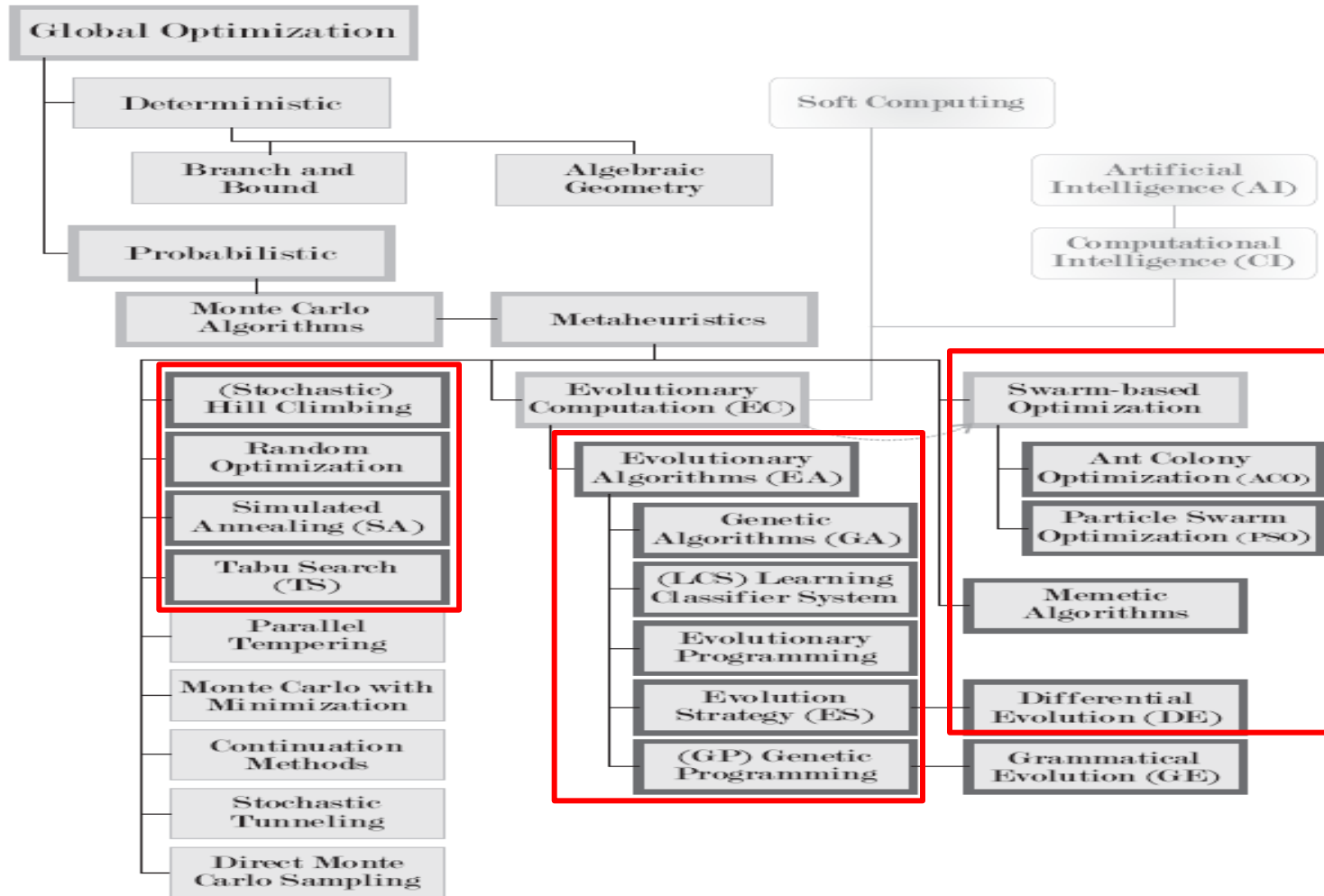
W.r.t. underlying metaphor:

- Biological evolution (e.g. evolutionary algorithms)
- Social interaction (e.g. swarm intelligence)
- Physics (e.g. simulated annealing)
- etc.



Metaheuristics Taxonomy [Wikipedia, nojhan.free.fr]

# Metaheuristics Taxonomy



# Structure of a generic trajectory based metaheuristic

S=initial candidate solution

Repeat

    R=modify(S)

    if quality(R)>quality(S) then R=S

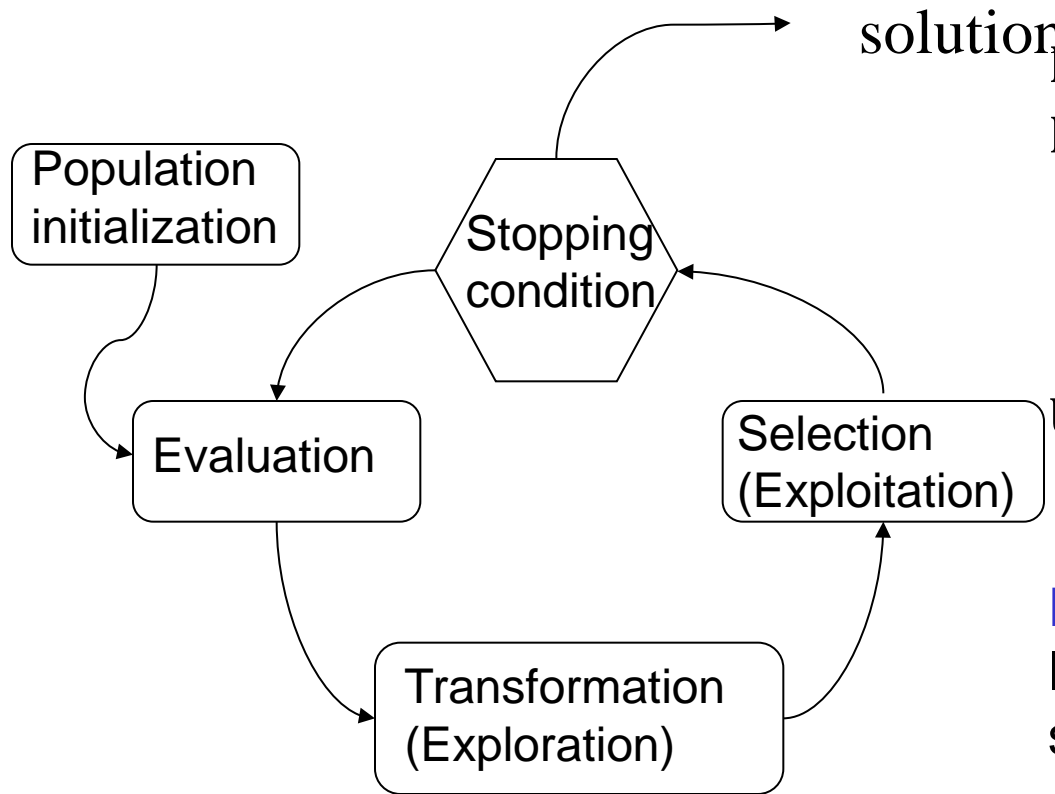
Until <stopping condition>

## Components:

- Initialization
- Modification (ensures the solution space exploration)
- Selection (ensures the exploitation of good quality solutions)

**Key element:** ensure the balance between exploration and exploitation

# Structure of a generic population based metaheuristic



$P$  = initial population

Repeat

evaluation of  $P$

$P' = \text{transformation}(P)$

$P = \text{selection}(P')$

Until <stopping condition>

**Population based metaheuristics =**

Iterative process based on successive application of

- exploration

- exploitation

operators on a randomly initialized population

# Topics

- Metaheuristics which use a candidate solution (trajectory-based) – Lectures 2-3
  - Pattern Search (Hookes Jeeves)
  - Nelder Mead
  - Random search (e.g Matyas, Solis Wets)
  
  - Restarted Local Search
  - Iterated Local Search
  - Simulated Annealing
  - Tabu Search
  - Scatter Search
  - Variable Neighborhood Search
  - Greedy Randomized Adaptive Search

# Topics

- Metaheuristics which use a population of solutions (population-based)
  - Evolutionary algorithms: (lectures 4-6)
    - ES – evolution strategies,
    - EP- evolutionary programming
    - GA – genetic algorithms
    - GP – genetic programming
  - Swarm intelligence algorithms: (lecture 7)
    - PSO - Particle Swarm Optimization,
    - ACO - Ant Colony Optimization
    - ABC - Artificial Bee Colony etc
  - Differential Evolution (lecture 8)
  - Algorithms based on estimation of the probability distribution (lecture 8)
    - PBIL – Population Based Incremental Learning
    - EDA – Estimation of Distribution Algorithms



# Course structure

- Scalable metaheuristic algorithms (Lecture 9)
  - Cooperative coevolution
  - Parallelization models:
    - Master-slave
    - Island model
    - Cellular model
- Algorithms for some classes of optimization problems (Lectures 10-12)
  - Constrained optimization
  - Multimodal optimization
  - Multi-objective optimization
  - Dynamic optimization
- Applications (Lectures 13-14)
  - Evolutionary design of neural networks
  - Data mining: feature selection, rules mining, clustering
  - Planning: routing, scheduling and resource allocation

# References

1. Sean Luke: *Essentials of Metaheuristics*, Lulu, second edition, 2013, available for free at <http://cs.gmu.edu/~sean/book/metaheuristics/>
2. Z. Michalewicz, D. Fogel: *How to Solve It. Modern Heuristics*. Springer, 1999
3. Jason Brownlee: *Clever Algorithms. Nature-inspired Programming Recipes*, 2011, available at <http://www.CleverAlgorithms.com>

# Lab

Lab 1: Introduction in SciLab and simple optimization problems

Lab 2: Random values in SciLab. Simulated Annealing, Tabu Search etc. Combinatorial optimization (TSP)

Lab 3: Evolutionary Algorithms. Continuous optimization

Lab 4: Swarm intelligence algorithms (PSO, ACO)

Lab 5: Multi-objective optimization - MOEA

Lab 6: Evolutionary design of neural networks

Lab 7: Data mining and planning applications

## Tools:

SciLab, R

# Evaluation

Course materials: <http://www.info.uvt.ro/~dzaharie/ma2018>

Evaluation:

Final project: 60-80%

Written test: 20%

Lab activity: 20%