## 03 Sequence alignment

### 03.04 Special-purpose alignment algorithms

- Overlap matches
- Local alignment
- Repeated matches


## Overlap matches

(allow ends to slide with no penalty)

- Align strings that either contain each other or overlap
- Similar to global alignment, but gaps at the two ends of each string are not penalized
- Alignment can start from any cell in the top or left border of the matrix $\quad F(0, j)=0$

$$
F(j, 0)=0
$$

- Alignment can end in any cell in the right or bottom border

$$
S(X, Y)=\max \left\{\begin{array}{l}
\max _{i} F(i, N) \\
\max _{j} F(M, j)
\end{array}\right.
$$

- Recursive equation is unchanged


## Overlap matches

(allow ends to slide with no penalty)

Example: BLOSUM50, linear gap penalty $d=8$

GAWGHEE
PAW-HEA

|  |  | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{A}$ | $\mathbf{G}$ | $\mathbf{A}$ | $\mathbf{W}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{E}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 0 | $\mathbf{0}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\mathbf{P}$ | 0 | -2 | -1 | -1 | -2 | -1 | -4 | -2 | -2 | -1 | -1 |
| $\mathbf{A}$ | 0 | -2 | -2 | 4 | -1 | $\mathbf{3}$ | -4 | -4 | -4 | -3 | -2 |
| $\mathbf{W}$ | 0 | -3 | -5 | -4 | 1 | -4 | $\mathbf{1 8}$ | $\mathbf{1 0}$ | 2 | 6 | -6 |
| $\mathbf{H}$ | 0 | 10 | 2 | 6 | -6 | -1 | 10 | 16 | $\mathbf{2 0}$ | 12 | 4 |
| $\mathbf{E}$ | 0 | 2 | 16 | 8 | 0 | 7 | 2 | 8 | 16 | $\mathbf{2 6}$ | 18 |
| $\mathbf{A}$ | 0 | -2 | 8 | 21 | 13 | 5 | 3 | 2 | 8 | 18 | $\mathbf{2 5}$ |
| $\mathbf{E}$ | 0 | 0 | 4 | 13 | 18 | 12 | 4 | 4 | 2 | 14 | $\mathbf{2 4}$ |

## Local alignment [Smith-Waterman, 1981]

Looks for best alignment between subsequences of $X$ and $Y$

- There are two main differences with respect to Needleman-Wunsch:

1. Negative scores are not allowed

$$
F(i, j)=\max \left\{\begin{array}{l}
F(l-1, j-1)+s\left(x_{i}, y_{j}\right) \\
F(i-1, j)-d \\
F(i, j-1)-d \\
0
\end{array}\right.
$$

1. The score of the best local alignment is the highest score in the matrix (that can be enywhere)

# Local alignment [Smith-Waterman, 1981] 

Example: BLOSUM50, linear gap penalty $d=8$

AWGHE
AW-HE

|  |  | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{A}$ | $\mathbf{G}$ | $\mathbf{A}$ | $\mathbf{W}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{E}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\mathbf{P}$ | 0 | 0 | 0 | 0 | $\mathbf{0}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\mathbf{A}$ | 0 | 0 | 0 | 5 | 0 | $\mathbf{5}$ | 0 | 0 | 0 | 0 | 0 |
| $\mathbf{W}$ | 0 | 0 | 0 | 0 | 2 | 0 | $\mathbf{2 0}$ | $\mathbf{1 2}$ | 0 | 0 | 0 |
| $\mathbf{H}$ | 0 | 10 | 2 | 0 | 0 | 0 | 12 | 18 | $\mathbf{2 2}$ | 14 | 6 |
| $\mathbf{E}$ | 0 | 2 | 16 | 8 | 0 | 0 | 4 | 10 | 18 | $\mathbf{2 8}$ | 20 |
| $\mathbf{A}$ | 0 | 0 | 8 | 21 | 13 | 5 | 0 | 4 | 10 | 20 | $\mathbf{2 7}$ |
| $\mathbf{E}$ | 0 | 0 | 0 | 13 | 18 | 12 | 4 | 0 | $\mathbf{4}$ | 16 | $\mathbf{2 6}$ |

## Repeated matches (asymmetric)

- $\quad$ Sequence $X$ contains a domain, or motif

We look for multiple local matches of $X$ in sequence $Y$, with score higher than a positive threshold $T$

$$
\begin{aligned}
& F(0, j)=\max \left\{\begin{array}{l}
F(0, j-1) \\
\max _{i}\{F(i, j-1)\}-T
\end{array}\right. \\
& F(i, j)=\max \left\{\begin{array}{l}
F(i-1, j-1)+s\left(x_{i}, y_{j}\right) \\
F(i-1, j)-d \\
F(i, j-1)-d \\
F(0, j)
\end{array}\right.
\end{aligned}
$$

## Repeated matches

HEAGAWGHEE
HEA. AW-HE .
Example: BLOSUM50, linear gap penalty $d=8, T=20$

|  |  | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{A}$ | $\mathbf{G}$ | $\mathbf{A}$ | $\mathbf{W}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{E}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 0 | 0 | $\mathbf{1}$ | 1 | 1 | 1 | 1 | 3 | $\mathbf{9}$ | $\mathbf{9}$ |
| $\mathbf{P}$ | 0 | 0 | 0 | 0 | $\mathbf{1}$ | 1 | 1 | 1 | 1 | 3 | 9 |  |
| $\mathbf{A}$ | 0 | 0 | 0 | 5 | 1 | $\mathbf{6}$ | 1 | 1 | 1 | 3 | 9 |  |
| $\mathbf{W}$ | $\mathbf{0}$ | 0 | 0 | 0 | 0 | 2 | 1 | $\mathbf{2 1}$ | $\mathbf{1 3}$ | 5 | 3 | 9 |
| $\mathbf{H}$ | 0 | $\mathbf{1 0}$ | 2 | 0 | 1 | 1 | 13 | 19 | $\mathbf{2 3}$ | 15 | 9 |  |
| $\mathbf{E}$ | 0 | $\mathbf{2}$ | $\mathbf{1 6}$ | 8 | 1 | 1 | 5 | 11 | 19 | $\mathbf{2 9}$ | $\mathbf{2 1}$ |  |
| $\mathbf{A}$ | 0 | 0 | 8 | $\mathbf{2 1}$ | 13 | 6 | 1 | 5 | 11 | 21 | $\mathbf{2}$ |  |
| $\mathbf{E}$ | 0 | 0 | 0 | 13 | 18 | 12 | 4 | 1 | 5 | 17 | 27 |  |

## Comparing alignments

Global alignment


Overlap


Local alignment


Repeated matches


# 03 Sequence alignment 

### 03.05 Heuristic algorithms

- Problem statement
- BLAST
- FASTA


## Heuristics

- The issue of performance
- Dynamic programming requires $\mathrm{O}(M N)$ steps
- Data base size (e.g. TrMBL)
- 314,641,655 amino acids in november 2003
- Grows very fast
- Query length: 300 a.a.
- Complexity: $10^{11}$ cells
- Computation time (at $10^{7}$ cells per second): 3 hours per query
- Solution
- Heuristic techniques trading off accuracy for performance
- Return not-only the best match to compensate for inaccuracy


## BLAST

[Altschul et al., 1990]
Use short high-score matching words as seeds from which to extend the alignment

- Create a table of short words (i.e., $3 \mathrm{r}, 11 \mathrm{bp}$ ) that match the query with high score
- Scan the database searching for words in the table
- Extend the matching region (without gaps) stopping at the maximum scoring extension


## FASTA

[Pearson and Lipman, 1988]
Search for short high-score matching words arranged on the same diagonal

1. Use a lookup table to locate short matching words (i.e., $2 \mathrm{r}, 6 \mathrm{bp}$ )
2. Search for diagonals with many matching words (this can be done by sorting matches on the difference of their indices $i-j$ )
3. Extend the matching seeds in the best diagonals (without gaps)
4. Use gaps to join ungapped regions

## FASTA

[Pearson and Lipman, 1988]
Example: BLOSUM50, linear gap penalty $d=8$, seed_size $=1$, min_score $=6$

1. Locate seeds with over threshold score

|  |  | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{A}$ | $\mathbf{G}$ | $\mathbf{A}$ | $\mathbf{W}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{E}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\mathbf{P}$ | 0 | -2 | -1 | -1 | -2 | -1 | -4 | -2 | -2 | -1 | -1 |
| $\mathbf{A}$ | 0 | -2 | -1 | $\mathbf{5}$ | 0 | $\mathbf{5}$ | -3 | 0 | -2 | -1 | -1 |
| $\mathbf{W}$ | 0 | -3 | -3 | -3 | -3 | -3 | $\mathbf{1 5}$ | -3 | -3 | -3 | -3 |
| $\mathbf{H}$ | 0 | $\mathbf{1 0}$ | 0 | -2 | -2 | -2 | -3 | -2 | $\mathbf{1 0}$ | 0 | 0 |
| $\mathbf{E}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{6}$ | -1 | -3 | -1 | -3 | -3 | 0 | $\mathbf{6}$ | $\mathbf{6}$ |
| $\mathbf{A}$ | $\mathbf{0}$ | -2 | -1 | $\mathbf{5}$ | 0 | $\mathbf{5}$ | -3 | 0 | -2 | -1 | -1 |
| $\mathbf{E}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{6}$ | -1 | -3 | -1 | -3 | -3 | 0 | $\mathbf{6}$ | $\mathbf{6}$ |

## FASTA

[Pearson and Lipman, 1988]
Example: BLOSUM50, linear gap penalty $d=8$, seed_size $=1$, min_score $=6$
2. Find best diagonal with multiple supporting seeds

|  |  | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{A}$ | $\mathbf{G}$ | $\mathbf{A}$ | $\mathbf{W}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{E}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | $\mathbf{Q}$ | $\mathbf{Q}$ | $\mathbf{Q}$ | $\mathbf{Q}$ | 0 | 0 | 0 | 0 | 0 |  |
| $\mathbf{P}$ | 0 | -2 | -1 | -1 | -2 | -1 | -4 | -2 | -2 | -1 | -1 |  |
| $\mathbf{A}$ | 0 | -2 | -1 | 5 | $\mathbf{Q}$ | 5 | -3 | $Q$ | -2 | -1 | -1 |  |
| $\mathbf{W}$ | $\mathbf{Q}$ | -3 | -3 | -3 | -3 | -3 | 15 | -3 | -3 | -3 | -3 |  |
| $\mathbf{H}$ | 0 | 10 | 0 | -2 | -2 | -2 | -3 | -2 | 10 | $\mathbf{Q}$ | 0 |  |
| $\mathbf{E}$ | $\mathbf{Q}$ | 0 | $\mathbf{6}$ | -1 | -3 | -1 | -3 | -3 | $\mathbf{Q}$ | $\mathbf{6}$ | $\mathbf{6}$ |  |
| $\mathbf{A}$ | 0 | -2 | -1 | 5 | 0 | $\mathbf{5}$ | -3 | 0 | -2 | -1 | -1 | 6 |
| $\mathbf{E}$ | 0 | 0 | $\mathbf{6}$ | -1 | -3 | -1 | -3 | -3 | 0 | $\mathbf{6}$ | $\mathbf{6}$ | 16 |

## FASTA

[Pearson and Lipman, 1988]
Example: BLOSUM50, linear gap penalty $d=8$, seed_size $=1$, min_score $=6$
3. Maximum ungapped extension

|  | $\mathbf{H}$ | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{A}$ | $\mathbf{G}$ | $\mathbf{A}$ | $\mathbf{W}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{E}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 0 | $\mathbf{Q}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\mathbf{P}$ | 0 | -2 | -1 | -1 | -2 | -1 | -4 | -2 | -2 | -1 | -1 |
| $\mathbf{A}$ | 0 | -2 | -1 | $\mathbf{5}$ | 0 | $\mathbf{5}$ | -3 | 0 | -2 | -1 | -1 |
| $\mathbf{W}$ | 0 | -3 | -3 | -3 | -3 | -3 | $\mathbf{1 5}$ | -3 | -3 | -3 | -3 |
| $\mathbf{H}$ | 0 | $\mathbf{1 0}$ | 0 | -2 | -2 | -2 | -3 | -2 | $\mathbf{1 0}$ | 0 | 0 |
| $\mathbf{E}$ | 0 | 0 | $\mathbf{6}$ | -1 | -3 | -1 | -3 | -3 | $\mathbf{0}$ | $\mathbf{6}$ | $\mathbf{6}$ |
| $\mathbf{A}$ | $\mathbf{0}$ | -2 | -1 | $\mathbf{5}$ | 0 | $\mathbf{5}$ | -3 | 0 | -2 | -1 | -1 |
| $\mathbf{E}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{6}$ | -1 | -3 | -1 | -3 | -3 | 0 | $\mathbf{6}$ | $\mathbf{6}$ |

## FASTA

[Pearson and Lipman, 1988]
Example: BLOSUM50, linear gap penalty $d=8$, seed_size $=1$, min_score $=6$
4. Possibly add gaps

|  |  | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{A}$ | $\mathbf{G}$ | $\mathbf{A}$ | $\mathbf{W}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{E}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}$ | 0 | 0 | 0 | $\mathbf{Q}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\mathbf{P}$ | 0 | -2 | -1 | -1 | -2 | -1 | -4 | -2 | -2 | -1 | -1 |
| $\mathbf{A}$ | 0 | -2 | -1 | $\mathbf{5}$ | 0 | $\mathbf{5}$ | -3 | 0 | -2 | -1 | -1 |
| $\mathbf{W}$ | 0 | -3 | -3 | -3 | -3 | -3 | $\mathbf{1 5}$ | -8 | -3 | -3 | -3 |
| $\mathbf{H}$ | 0 | $\mathbf{1 0}$ | 0 | -2 | -2 | -2 | -3 | -2 | 10 | 0 | 0 |
| $\mathbf{E}$ | 0 | 0 | $\mathbf{6}$ | -1 | -3 | -1 | -3 | -3 | 0 | $\mathbf{6}$ | $\mathbf{6}$ |
| $\mathbf{A}$ | $\mathbf{0}$ | -2 | -1 | $\mathbf{5}$ | 0 | $\mathbf{5}$ | -3 | 0 | -2 | -1 | -1 |
| $\mathbf{E}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{6}$ | -1 | -3 | -1 | -3 | -3 | 0 | $\mathbf{6}$ | $\mathbf{6}$ |

