Applied Soft Computing 11 (2011) 489–499

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Artificial Bee Colony (ABC) for multi-objective design optimization of composite structures

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ARTICLE INFO

Article history:
Received 18 August 2008
Received in revised form 4 November 2009
Accepted 6 December 2009
Available online 16 December 2009

Keywords:
Swarm intelligence
Artificial Bee Colony (ABC)
Vector Evaluated Artificial Bee Colony (VEABC)
Composite
Structural optimization
Multi-objective optimization

ABSTRACT

In this paper, we present a generic method/model for multi-objective design optimization of laminated composite components, based on Vector Evaluated Artificial Bee Colony (VEABC) algorithm. VEABC is a parallel vector evaluated type, swarm intelligence multi-objective variant of the Artificial Bee Colony algorithm (ABC). In the current work a modified version of VEABC algorithm for discrete variables has been developed and implemented successfully for the multi-objective design optimization of composites. The problem is formulated with multiple objectives of minimizing weight and the total cost of the composite component to achieve a specified strength. The primary optimization variables are the number of layers, its stacking sequence (the orientation of the layers) and thickness of each layer. The classical lamination theory is utilized to determine the stresses in the component and the design is evaluated based on three failure criteria: failure mechanism based failure criteria, maximum stress failure criteria and the tsai-wu failure criteria. The optimization method is validated for a number of different loading configurations—uniaxial, biaxial and bending loads. The design optimization has been carried for both variable stacking sequences, as well fixed standard stacking schemes and a comparative study of the different design configurations evolved has been presented. Finally the performance is evaluated in comparison with other nature inspired techniques which includes Particle Swarm Optimization (PSO), Artificial Immune System (AIS) and Genetic Algorithm (GA). The performance of ABC is at par with that of PSO, AIS and GA for all the loading configurations.

1. Introduction

Now-a-days composites are becoming increasingly popular, due to their superior mechanical characteristics, like very high stiffness to weight ratios and amenability to tailoring of these properties. Remarkable variations in the characteristics of composite materials can be achieved by slightly altering their properties. Thus, composite materials offer the possibility to create an unlimited set of different material behaviors that can be tailored to specific structural needs. The use of laminates increases the freedom in design and gives more control to fine-tune the material to meet local design requirements. However, the analysis and design of composite materials is relatively more complex. Composite design optimization typically consists of identifying the optimal configuration that would achieve the required strength with minimum overheads. The possibility to achieve an efficient design that fulfills the global criteria and the difficulty to select the values out of a large set of constrained design variables makes mathematical optimization a natural tool for the design of laminated composite structures [1]. Depending on the nature of application for which the component is being designed, there would be a number of different overheads like weight, cost, etc which have to be taken into consideration for effective design optimization of composites. Thus, making this problem multi-objective in nature. There has been considerable amount of work carried out on composites' design optimization [1–7]. Laminate stacking sequence design optimization has been formulated as a continuous optimization problem and solved using various gradient based methods by Gürdal and Haftka [2]. Bruyneel [3] has presented a general and effective procedure based on a mathematical programming approach for the optimal design of composite structures subjected to weight, stiffness and strength criteria. Shin et al. [4] have investigated the minimum-weight design of simply supported, symmetrically laminated, thin, rectangular, especially orthotropic laminated plates for buckling and post-buckling strengths. Adali et al. [5], Kumar and Tauchert [6] and Pelletier et al. [7] have discussed the multi-objective design of symmetrically laminated plates for different criteria like strength, stiffness and minimal mass. Venkataraman and Haftka [8] have presented a review of various approaches to the optimization of composite panels.

Composite laminate design problems typically involve multi-modal search spaces [8] with the design variables capable of taking

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doi:10.1016/j.asoc.2009.12.008

Nomenclature

a, b, h length, width, thickness of the plate
E_{LL}, E_{TT}, E_{tt} longitudinal, transverse, normal elastic moduli
G_{LT}, G_{Lt}, G_{Tt} shear moduli in longitudinal, lateral and traverse directions
N, α, β, γ number of artificial bees, randomness amplitude of bee, convergence rate, learning rate
$N_x, N_y, N_{xy}, M_x, M_y, M_{xy}$ stress resultants, moment resultants in X–Y plane
S shear strength in the X–Y plane
t_k, θ, N ply thickness, stacking sequence, number of plies
$\upsilon_{LT}, \upsilon_{Lt}, \upsilon_{Tt}$ lamina Poisson's ratio's in longitudinal, lateral and traverse directions

Greek symbol

σ_{LL}, σ_{TT}, σ_{LT} longitudinal, transverse, shear stress along material axes

a wide range of values, making this a combinatorially explosive problem. For such problems, traditional gradient based algorithms are plagued with the problem of converging to locally optimal regions of the design space. Multi-objective design of composites warrants the use of modern non-parametric optimization methods.

In pursuit of finding solution to these problems many researchers have been drawing inspiration from the nature. A host of such nature inspired techniques have been developed namely Genetic Algorithm (GA) [9], Artificial Neural Networks (ANN) [10], Particle Swarm Optimization (PSO) [11,12] and Artificial Immune System (AIS) [13]. These algorithms with their stochastic means are well equipped to handle such problems. Of these, GA has been a very popular tool for the combinatorial design optimization of composite structures [9]. It has been extensively employed for the optimal structural design and there are numerous successful implementations available in the literature for the same [9,14–19].

In the current work, we present a generic multi-objective approach for the composite design optimization based on another such, relatively new swarm intelligence technique—Artificial Bee Colony (ABC) [20]. Swarm intelligence algorithms are a new range of computational algorithms that have emerged from the behaviour of social insects. Social insects are usually characterized by their self-organization (in numerous situations the coordination arises from interactions among individuals) and the absence of central control. Still, complex group behaviour emerges from the interactions of individuals who exhibit simple behaviours by themselves. In social insects, every individual is autonomous. They can only obtain local information, and interact with their geographical neighbours. All these features characterize swarm intelligence. Examples of systems like this can be found in nature, including bee colonies, ant colonies, bird flocking, animal herding, fish schooling etc. Inspired by the bee behaviour, Artificial Bee Colony [20] is one of the generally applicable techniques used for optimizing numerical functions and real-world problems. Compared with GA and other similar evolutionary techniques, ABC has some attractive characteristics and in many cases proved to be more effective [20]. Both GA and ABC have been used extensively for a variety of optimization problems and in most of these cases ABC has proven to have superior computational efficiency [20,21]. Further, ABC does not use any gradient-based information. It incorporates a flexible and well-balanced mechanism to adapt to the global and local exploration and exploitation abilities within a short computation time. Hence, this method is efficient in handling large and complex search spaces. ABC with its ability to handle combinatorial explosive problems appears to be very promising for the multi-objective optimization problem addressed in this paper.

The multiple objectives considered here are—minimizing the weight of the composite component and also minimizing the total cost (manufacturing and material costs). The primary design variables are—number of layers, lamina thickness and the stacking sequence. These variables are altered so as to attain an optimum composite design that achieves both the above mentioned objectives while satisfying the specified strength requirements. In the current work, the stacking sequence is not restricted to the popularly used schemes like {0/45/90}, {0} and {0/90}. Instead, the ply orientation angles are also considered as variables of the optimization process, thereby allowing for evolving new non-standard stacking schemes, appropriate for the specified application. This ensures a truly optimal design for the given application as all the possible stacking sequences are explored. The classical lamination theory is utilized to determine the stresses at each layer for thin laminates subjected to force and/or moment resultants and the design is evaluated based on the specified failure criteria. The use of appropriate failure criteria is crucial for the optimal design of composite laminates. Since different failure mechanisms are relevant for different loading combinations, in the current work we evaluate the composite design for three different failure criteria; Tsai-Wu [22], Maximum Stress [1] and the Failure Mechanism based criteria [23]. This makes the optimization method truly generic and ensures a completely optimum solution/configuration for the given application.

The generic composite design optimization framework being presented in the current work employs Vector Evaluated Artificial Bee Colony (VEABC), a variant of the classical ABC for multi-objective optimization. This method allows for separate evaluation of the multiple objectives, which proves to be very appropriate for the current problem. This is a swarm intelligence method which employs separate swarms for each of the objectives and information migration between these swarms ensures an optimal solution with respect to all the objectives.

This paper is structured as follows: basics of multi-objective problems are presented in Section 2. Section 3 introduces ABC and VEABC. Details of the problem and its formulation are explained in Section 4. The outline of the optimization process employed is given in Section 5. The numerical results and discussions are presented in Section 6. Finally, the comparison of nature inspired techniques and conclusions are given in Sections 7 and 8 respectively.

2. Multi-objective optimization

Let X be a n-dimensional search space, and $f_i(x)$, $i = 1 \ldots k$, be k objective functions defined over X. Furthermore, let $g_i(x) \leq 0$, $i = 1, \ldots, m$, be m inequality constraints. Then, the multi-objective problem can be defined as finding a vector, $x = (x_1, x_2 \ldots x_n)^T \in X$ that satisfies the constraints, and optimizes the vector function,

$$f(x) = \{f_1(x), f_2(x), \ldots, f_k(x)\}^T. \tag{1}$$

In the case of multi-objective problems the concept of *Pareto optimality* [24,25] is introduced. A solution x of the multi-objective problem is said to be *Pareto optimal* if and only if there does not exist another solution y, such that $\mathbf{f}(y)$ dominates $\mathbf{f}(x)$. The objective functions $f_i(x)$, may be conflicting with each other, thus, most of the time it is impossible to obtain for all objectives the global minimum at the same point. Instead there exists a set of optimal trade-offs which forms the solution set—the *Pareto set* and it is denoted by P^*. The set $PF^* = \{\mathbf{f}(x) | x \in P^*\}$ is called the *Pareto front*.

3. Artificial Bee Colony

In social insect colonies, each individual seems to have its own agenda; and yet the group as a whole appears to be highly organized. The algorithms based on swarm intelligence and social insects begin to show their effectiveness and efficiency to solve difficult problems [9–13]. A swarm is a group of multi-agent system such as bees, in which simple agents coordinate their activities to solve the complex problem of the allocation of labor to multiple forage sites in dynamic environments. An important and interesting behavior of bee colonies is their foraging behavior, and in particular, how bees find a food source based on the amount of nectar and successfully bring nectar back to the hive. In a real bee colony the bees are grouped as scout bees, employed bees and onlookers. Initially, the foraging process begins in a colony by scout bees (unemployed bees) which explore food sources by moving randomly. At the entrance of the hive is an area called the dance-floor, where dancing takes place. Upon their return to the hive from a foraging trip, it communicates by performing the so-called waggle dance [26] so as to recruit other bees to go to the food source. A bee waiting on the dance area for making decision to choose a food source is called an onlooker, which seems to learn information from the dance regarding the food source: its nectar amount, the direction in which it will be found and its distance [26,27]. If the scouts discover rich food source then the scout bees are selected and classified as the forager bee (employed bee). After waggle dancing the forager bee leaves the hive to get nectar with their fellow bees that were waiting inside the hive. The number of follower bees assigned to nectar depends on the overall quality of the nectar. Upon arrival, the bees take a load of nectar and return to the hive relinquishing the nectar to a food-storer (onlooker) bee. In this way a good food source is exploited, and the number of foragers at this site is reinforced.

In a robust search process, exploration and exploitation process must be carried out together. In the ABC algorithm [20,21], the scout bees control the exploration process, while the employed bees and onlookers' carryout the exploitation process in the search space. The number of employed bees and the onlookers is equal to the total population. The employed bee whose food source has been exhausted becomes a scout bee. The position of an enhanced nectar amount of a food represents a possible solution to the optimization problem.

At the first step, create a population of n artificial bees placed randomly in the search space representing the food source position, where n denotes the size of population. After initialization, the population of the positions (solutions) is subjected to repeated iteration of the search processes of the employed bees, the onlooker bees and scout bees. This search process can be divided into two phases:

(i) Exploration phase

For each solution x_{ij}, where $i = 1,2\ldots n$ and j is dimensional vector. The scout bees explore a new food source with x_i. This operation can be defined as in (2)

$$x_i^j = x_{\min}^j + (x_{\max}^j - x_{\min}^j)\,\mathrm{rand}(0,1) \tag{2}$$

Here the value of each component in every x_i vector should be clamped to the range $[x_{\min}, x_{\max}]$ to reduce the likelihood of scout bees leaving the search space (S). The population spread is restricted within the search space S i.e $x_{ij} \in S$ and in Eq. (2) $x_{\min}$ and $x_{\max}$ is the lower and upper limit respectively of the search scope on each dimension.

(ii) Exploitation phase

In this phase, assuming the scout bees which have explored food source are selected as employed bees, which randomly perturb to the nearest neighbour, this produces a modification on the position (solution) in her memory depending on the local information (visual information) and tests the nectar amount (fitness value) of the new source (new solution). If the nectar amount of the new one is higher than that of the previous one, the bee memorizes the new position and forgets the old one. Otherwise it memorizes the position of the previous one. After all employed bees complete the search process; they communicate the nectar information of the food sources and their position information with the onlooker bees on the dance area. An onlooker bee evaluates the nectar information taken from all employed bees and chooses a food source with better nectar amount. As in the case of the employed bee, onlooker bee also produces a modification on the position in her memory and checks the nectar amount of the candidate source. Providing that its nectar is higher than that of the previous one, the bee memorizes the new position and forgets the old one.

An artificial onlooker bee chooses a food source depending on the new positions, using Eq. (3).

$$P_i = \begin{cases} v_i, & \text{if}(f(x_i) \geq f(v_i)) \\ x_i, & \text{if}(f(x_i) \leq f(v_i)) \end{cases} \tag{3}$$

In order to select the better nectar position found by an onlooker, O_b is defined as

$$O_b = \arg\min_{P_i} f(P_i), \quad 1 \leq i \leq n \tag{4}$$

where P_i is the best fitness value of the solution i which is proportional to the nectar amount of the food source in the position i and n is the number of food sources which is equal to the number of employed bees.

In order to produce a candidate food position from the old one in memory, the ABC uses the following Eq. (5):

$$v_{ij} = x_{ij} + \alpha(x_{ij} - x_{kj}) \tag{5}$$

where $k = 1, 2,\ldots,n$ and $j = 1, 2,\ldots,D$ are randomly chosen indexes. Although k is determined randomly, it has to be different from i. α is an adaptively generated random number. It controls the production of neighbour food sources around x_{ij} and represents the comparison of two food positions visually by a bee. As can be seen from (5), as the difference between the parameters of the x_{ij} and x_{kj} decreases, the perturbation on the position x_{ij} gets decreased, too. Thus, as the search approaches the optimum solution in the search space, the step length is adaptively reduced. The food source of which the nectar is abandoned by the bees is replaced with a new food source by the scout bees. A brief description of ABC algorithm is given below

ALGORITHM: A High-Level Description of ABC

1. Create a initial population of artificial bees within the search space x_{ij}
2. Evaluate the fitness of the population
3. While (stopping criterion not met)

(i) Produce new solutions (food source positions) v_{ij} in the neighbourhood of x_{ij} for the employed bees using Eq. (5)
(ii) Evaluate the fitness value and apply the selection process between x_{ij} and v_{ij} using Eqs. (3) and (4)
(iii) Produce new solutions (new positions) v_{ij} for the onlookers from the solutions x_{ij}, selected depending on P_i and evaluate them
(iv) Determine the abandoned solution (source) x_{ij}, if exists, and replace it with a new randomly produced solution x_{ij} for the scout bee using Eq. (2)

(v) Memorize the best food source position (solution) achieved so far
4. End while

There have been several other recent methodologies proposed for solving optimization problems based on the inspiration from the way honey bees forage for food. They are as follows: (i) the multi-objective optimization using the Bees Algorithm proposed by Pham and Ghanbarzadeh [28]; (ii) the virtual bee colony by Yang [21]; (iii) the Bee Colony Optimization (BCO): principles and applications by Teodorovic et al. [29].

The algorithm used in this paper is based on the Parallel Vector Evaluated Artificial Bee Colony, a multilevel bee variant of ABC, which is inspired by the Vector Evaluated Genetic Algorithm (VEGA) [30] and Vector Evaluated Particle Swarm Optimization (VEPSO) [11,12]. It can be observed from the literature that this approach to composite design optimization has not yet been extensively explored.

3.1. Vector Evaluated ABC—VEABC

The VEABC is a multi-objective ABC method inspired by the concept and main ideas of VEGA algorithm [30] and VEPSO algorithm [11,12]. The VEABC algorithm is conceptually simple. It is similar to two single objective functions being separately evaluated by separate artificial bees. The multiple objectives being considered here are disparate in nature and hence this renders the separate/exclusive evaluation of the multiple objectives more appropriate. VEABC is very well suited for the current problem, as it is capable of searching for multiple optimal solutions in a very vast solution space, in a single run using swarm intelligence techniques.

The key issue in these swarm intelligence algorithms is that the fitness of an individual in a population depends on individuals of a different population. This enhances the capability of the algorithm to better explore and exploit the search space, thereby more accurately detecting the convex, concave or partially convex and/or concave and/or discontinuous Pareto fronts [30,31]. The main features of VEABC are explained in detail below:

The Vector Evaluated method assumes that M swarms D_1, $D_2 \ldots D_M$ each of size n aim to optimize simultaneously M-objective functions. Each swarm is exclusively evaluated according to one of the objective functions. In VEABC, a population of n artificial bees of M swarms placed randomly in the search space, evaluate the fitness of the best food source visited by artificial bees, and then from each swarm select bees that have highest fitness as forage bees. Update to new position (solution) after the recruitment is managed by the forager bees. The best solution is obtained after certain time of progression based on the number of bees visiting the same location.

At each time step for M swarms each of size n artificial bees; the distance (randomness amplitude) and direction (convergence rate) of each bees is changed towards its most favourable position (solution). The random factor prevents the swarm getting stuck in the wrong place and speed of convergence is used to identify the rate at which bees converge to a solution.

For a given M swarm every bee continuously updates itself towards the above mentioned best solution. Thus a new generation of community comes into being, which has moved closer towards a better solution, ultimately converging onto the optimal solution. In practical operation, for the scale of n number of artificial bees, the current location of all the artificial bees is expressed as ${}^{[j]}D_i(t)$ at a given time-t for jth swarm. The fitness function, which is determined by the optimization problem, assesses the extent of most favourable position (solution). The best position of the whole population which gave the overall best value of the fitness function is *Dbest*. Then the VEABC's swarms should be updated according to Eq. (6).

$$ {}^{[j]}D_{(i+1)} = \alpha(r-S) + (1-\beta)^{[j]}D_i + \gamma^{[k]}\mathrm{Dbest}_i \quad (6) $$

Here the superscripts represent the ABC parameters for the jth swarm. Here α is the *randomness amplitude of bee*, β is the *convergence rate* and γ is the *learning rate*. The factor r is randomly generated within the range $\{0, 1\}$ and S is the step size. The VEABC assumes that the search behavior of a swarm is affected by a neighboring swarm—kth swarm. The parameter k can be selected in a number of ways, resulting different information migration schemes between the multiple swarms. The equation is essentially made up of three parts. The first and second parts are the distance and direction of the bees, which gives the convergence rate and randomness amplitude of the bee. The third part is the best estimates, which expresses the most favourable position (solution). These three parts together determine the solution space searching ability. The first and second parts cause the bees to search the whole and avoid local minimum. The third part reflects the information sharing by forage bee related to most favourable position (solution) to their fellow bees. Subsequently the bees reach an effective and best position.

4. The optimization problem—problem formulation

The current problem has been framed as a multi-objective optimization problem of having to minimize both the weight of the component as well as the total cost for a composite for required strength, so that it satisfies the specified failure criteria. The decision variables considered are the number of layers, stacking sequence and the lamina thickness. It has been clearly illustrated in the earlier sections that these variables have very wide ranges and are associated with a number of different constraints. Thus, this problem falls under the class of constrained non-linear optimization problem with a vast solution space. Generally constrained non-linear optimization problems (CNOP) are made up of three basic components; a set of variables, objective function(s) to be optimized and a set of associated constraints that define the feasible solution space. The goal is to find the values of the variables within the feasible space that optimizes the objective function(s) while satisfying the constraints. ABC has been successfully used for many standard optimization problems and has established itself as a very effective optimization tool [20]. However, its application in multi-objective and constrained optimization problems is a new area.

4.1. Structural analysis model

In the current study, the generic model given in [23] is used for the design of the composite laminate. Here, expressions of classical laminated plate theory (CLPT) are employed for designing of the composite laminates with desired in-plane stiffness properties. Laminates symmetric about the mid-plane are considered for the design and weight estimation. The stress resultants (N_x, N_y, N_{xy}) are obtained for the laminate subjected to in-plane stresses using the expression

$$ \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{61} & A_{66} \end{bmatrix} \{\varepsilon\} \quad (7) $$

where $\{\varepsilon\}$ is the strain matrix and the coefficients of $[A]$ are given as,

$$ A_{ij} = \sum_{k=1}^{N} (\bar{Q}_{ij})_{(k)} (Z_k - Z_{k-1}) \quad (8) $$

$(\bar{Q}_{ij})_{(k)}$ are the transformed reduced stiffness of the kth ply and the $(Z_k - Z_{k-1})$ represent the thickness of the corresponding kth ply.

The description of the three failure criteria, Tsai-Wu failure criteria [22], Failure Mechanism Based failure criterion [23] and Maximum Stress failure criterion and the design constraints are given by Omkar et al. in [11,13].

4.2. Objective functions

The objectives considered for the design optimization are two fold; firstly to minimize the weight of the composite component and secondly to minimize the total cost involved (material costs + manufacturing costs).

4.2.1. The weight function

The procedure for the determination of optimum weight of the laminate is as follows. For a given loading condition, the design variables are ply thickness, stacking sequence and the number of plies. Reduced stiffness and compliance matrices are obtained for the given material properties. The strains, curvatures, global stresses and material ply stresses are then obtained. Finally, failure condition of the ply is obtained by checking each ply stress condition with the strength of the lamina. The optimum weight of the laminate is calculated such that all plies satisfy the failure condition. Total width of the composite laminate with $n_{\theta i}$ representing the number of layers at orientation angle θ_i with thickness of t mm is given by;

$$h = [\{\sum_{i=1}^{12}(n_{\theta i})\}t] \tag{9}$$

$$\text{Weight}, \ W_t = \rho hab \tag{10}$$

where ρ is the density of the material of the composite laminate

4.2.2. The cost function

Considering the economics is a very important design objective. This is very aptly justified due to the high costs of the composite materials. The component is optimized with respect to minimum cost 'g', which can be formulated as the sum of the material and manufacturing costs. The cost function developed by Kovacs et al. [32] for carbon-fibre-reinforced plastic (CFRP) sandwich-like structure with aluminium (Al) is used,

Total cost = material cost + manufacturing cost

$g = g_{\text{matl}} + g_{\text{manufact}}$

$$g(x) = [g_{\text{matl}}\{W_t\} + 4 + g_{\text{maf}}\{(\sum_{i=1}^{12}(n_{\theta i}))14_{\text{min}} + 110_{\text{min}}\}]\$ \tag{11}$$

The main contribution to the material cost arises from the raw material for the composite plates. The manufacturing cost is a direct function of time (in minutes) associated with manufacturing of the laminates, which includes the time lost in press form preparation, layer cutting, layer sequencing and final working. The indices g_{matl} and g_{maf} are determined based on the material being used and the type of manufacturing process employed.

4.3. Decision variables and constraints

The definition of a directional laminate composite requires the specification of the fibre direction, the number of layers and the thickness of each layer. Hence, these form the design variables of the laminate design optimization process.

The stacking sequence describes the orientation of the plies i.e. the number of plies placed at different orientation angles. This stacking sequence has a pronounced effect on the properties of the composites and greatly affects the strength of the composite component. In the current work, fibre orientation angles are considered within the range of $\{-75^\circ, +90^\circ\}$ in steps of 15°, hence 12 different possible values of orientation angle θ;

$$\theta = \{-75^\circ/-60^\circ/-45^\circ/-30^\circ/-15^\circ/0^\circ/15^\circ/30^\circ/45^\circ/60^\circ/75^\circ/90^\circ\} \tag{12}$$

Here we have considered the number of layers present at each of the different fiber orientation angles as the actual decision variables for the optimization process. The second decision variable—the thickness of the lamina, has been considered to be within the range of 0.05–0.5 mm. In this case, we have assumed each layer to be of the same thickness i.e. $\{t_1/t_2/,\ldots,/t_i\} = t$.

The minimum value of weight and total cost is achieved by determining the optimal configuration, given by,

$$\{n_{\theta 1}/n_{\theta 2}/,\ldots,/n_{\theta i}\}_{\text{sym}} \quad \{t_1/t_2/,\ldots,/t_i\} \tag{13}$$

where, $n_{\theta i}$ is the number of layers at a fibre orientation angle θ_i and t_i is the thickness of ith layer of the laminate. In the current problem, we have a total of thirteen decision variables. Twelve variables $\{n_{\theta 1}/n_{\theta 2}/,\ldots,/n_{\theta 12}\}$ corresponding to the number of layers at each of the twelve different fiber orientation angles. Further, the plies at different fiber orientation angles are discrete variables capable of only integer values within the specified range. Also, the lamina thickness is assumed as a discrete variable in view of retaining the practicality of the solution evolved. The lamina thickness had been constrained such that it is only capable of taking values within the specified range with a minimum increment of 0.001 mm.

In laminated composite structures, ply thickness (t), number of plies (N) and orientation angles (y) are the major variables. These variables contribute to the major strength and stiffness of the laminate. Hence, these/only three design variables are considered for the design optimization of laminated composite structures.

Mathematically,

Minimize weight: Min$\{w = f(x)\}$, $f(x)$ *weight objective function*

Minimize cost: Min$\{c = g(x)\}$, $g(x)$ *cost objective function*

Such that, strength > minimum strength: generic failure criterion,

Strengths are calculated using the reference structural mathematical model [33].

A possible solution, $x = \{[n_{\theta 1}/n_{\theta 2}/\ldots/n_{\theta i}], t\}$

Number of layers at each orientation angle, $n_{\theta i} \forall n_{\theta i} \in Z^+ \ni Z^+ = [0,\ldots,50]$

Also,

$$\sum_{i=1}^{12}(n_{\theta i}) > 0 \forall \theta_i \in \theta \ni \theta = [-90 + (15i)]^{\text{o}} \quad \text{for } i = 1\ldots 12. \tag{14}$$

Thickness, $t \in S$; $S = [0.05\text{ mm}, 0.5\text{ mm}]$.

5. The optimization process

A carbon/epoxy composite laminate is considered in current work. The material properties considered for the study are given in Table 1. The laminate subjected to in-plane loadings are shown in Fig. 1.

Here, the ABC method is incorporated with the necessary modifications to render it applicable to constrained non-linear optimization problems with discrete design variables. The key point in the constrained optimization process is dealing with the constraints associated with decision variables. In the current work, the constraints are effectively handled by preserving the feasibility of the solutions evolved. In order to constrain the optimum solution to the feasible space, each artificial bee is made to search the entire solution space but keeping track of only the feasible solutions as it progresses. Further, this process is accelerated by initializing the

Table 1
Mechanical properties of the carbon fibre reinforced plastic lamina (AS4 and Epoxy 3501-6) (moduli are in GPa).

Elastic moduli of laminate			Lamina Poisson's rations			Rigidity moduli of laminate		
E_{LL} Longitudinal	E_{TT} Transverse	E_{tt} Normal	ν_{tL} tL direction	ν_{LT} LT direction	ν_{Tt} Tt direction	G_{LT} LT direction	G_{Tt} Tt direction	G_{tL} tL direction
126	11	11	0.28	0.28	0.4	6.6	3.93	6.6

artificial bees within the feasible solution space. The design variables involved in the current optimization problem are discrete in nature. The ABC algorithm used in the current work is modified to handle discrete variables. The twelve variables $[n_{\theta1}/n_{\theta2}/,\ldots,/n_{\theta12}]$ corresponding to the number of layers at each of the twelve different fiber orientation angles are capable of taking only integer values within the specified range making them discrete in nature. Further, the lamina thickness—t is also considered to be a discrete variable, capable of taking values between the specified ceiling and floor limits with a least count of 0.001 mm. This consideration has been taken in view of retaining the practicality of the evolved solution in terms of its manufacturability.

VEABC employs two or more swarms to probe the search space and information is exchanged among them. Each swarm is exclusively evaluated with one of the objective functions, but, information coming from other swarm(s) is used to influence its motion in the solution space. Thus, exchanging this information among swarms leads to Pareto optimal points. Specifically, in this case since there are two objective functions, two swarms (X_1, X_2) of N artificial bees each are used. X_1 evaluates the *weight objective function* and X_2 evaluates the *cost objective function*. There is no necessity for a complicated information migration scheme between the swarms as only two swarms are employed. Each swarm is exclusively evaluated according to the respective objective function. The most favourable position (solution) of the second swarm (X_2) is used for calculation of the new favourable position (solution) of the first swarm (X_1) and accordingly the most favourable position (solution) of the first swarm (X_1) is used for calculation of the new favourable position (solution) of the second swarm (X_2).

The artificial bee's position updates equations for the first swarm—X_1

$$^{[X_1]}D_{(i+1)} = \alpha(r - S) + (1-\beta)^{[X_1]}D_i + \gamma^{[X_2]}\mathrm{Dbest}_i \quad (15)$$

The artificial bee's position updates equations for the second swarm—X_2

$$^{[X_2]}D_{(i+1)} = \alpha(r - S) + (1-\beta)^{[X_2]}D_i + \gamma^{[X_1]}\mathrm{Dbest}_i \quad (16)$$

The particles of both the swarms (X_1, X_2) move in solution space according to the above mentioned equations, successively aligning themselves with respect to both the objective functions in each iteration and finally converging on the global optimum solution.

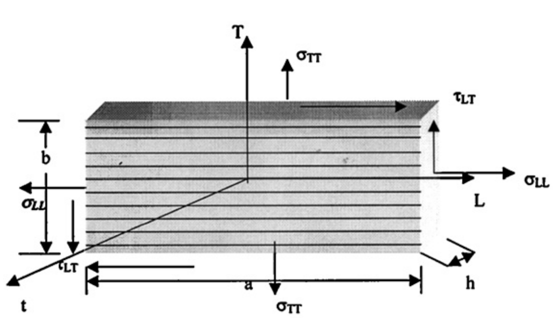

Fig. 1. A typical loading configuration of the composite component.

The performance of the ABC is very sensitive to the control parameter choices. The ABC parameters used in the current case are listed in Table 2. The number of artificial bees is decided by trial and error method. A number of simulations are carried out with different number of swarm particles and the optimal solution is observed when ten times the number of dimensions of the problem is taken as the number of artificial bees. As the current problem is 13-dimensional, 130 artificial bees are used for both the swarms. During initialization, it is ensured that all the artificial bees are within the feasible solution space, since randomly initialized artificial bees are not always confined to the feasible solution space. The ABC parameters are the same for each swarm and for all simulation runs.

The remaining parameters, *Randomness Amplitude of bee* α, the *Convergence rate* β and the *Learning rate* γ are adjusted dynamically during the optimization. A starting value of $\beta = 1$ is used to initially accommodate a more global search and is dynamically reduced to $\beta = 0$. The β value is adaptively allocated as per Eq. (17);

$$\beta = \beta_{\max} - [\{(\beta_{\max} - \beta_{\min})/\mathrm{it}_{\max}\}\mathrm{it}] \quad (17)$$

where $\beta_{\max}$ is the initial *Convergence rate* value, $\beta_{\min}$ is the final *Convergence rate* value, it is the current iteration number and $\mathrm{it}_{\max}$ is the maximum number of iterations. The initial higher value may result in greater population diversity in the beginning of the optimization, whereas at a later stage lower values are favoured, causing a more focused exploration of the search space. Similarly, *Randomness Amplitude of bee* α and the *Learning rate* γ are also adjusted dynamically.

Following is the overview of the algorithm of the modified VEABC employed for the current optimization problem.

The algorithm

1. Initialize both swarms (X_1, X_2) randomly within the feasible solution space.
   a. All artificial bees are repeatedly initialized until it satisfies all the constraints.
2. While the end condition is false.
3. For both the swarms exclusively evaluate each of the objectives.
   a. X_1: Evaluates the weight-objective function.
   b. X_2: Evaluates the cost-objective function.
4. If the fitness value is better for the current solution than the previous best solution, then assign the current solution as the best solution—for both the swarms.
5. Adaptively generate the Randomness Amplitude of bee, the Convergence rate and the Learning rate.
6. For both the swarms update the bee positions towards the most favourable position (solution) from the other swarm.
7. After each update check whether the variables of each artificial bee of both the swarms satisfy the constraints.
8. IF the number of iterations without updating the best value of both Swarms > maximum number of iterations THEN condition = true, End. Else the whole process is repeated (Back to 2).

Table 2
The ABC parameters.

Convergence rate	β = [1,. . .,0.4], adaptively allocated (decreasing from 1 to 0.4 with each iteration)
Learning rate	γ = [1,. . .,0.4], adaptively allocated (decreasing from 1 to 0.4 with each iteration)
Randomness amplitude of bee	α = [4,. . .,0], adaptively allocated (decreasing from 4 to 0 with each iteration)
Step size	S = 0.75
Number of swarm particles	N = 130
Maximum number of iterations	Max_it = 1000 iterations
End condition (number of iterations without update in the best values)	500 iterations

6. Results and discussions

The simulation studies have been carried out on specimen carbon/epoxy laminate plate with in-plane dimensions of a = 10 m, b = 10 m. L, T and t are designated as longitudinal, transverse, normal directions of the plate and a typical loading configuration of the plate is illustrated in Fig. 1. The physical properties of the unidirectional carbon/epoxy laminate material are listed in Table 1.

It is crucial to use the appropriate failure criteria to achieve an optimal design configuration [11,13]. Hence in this study, we have considered three different failure theories to arrive at a truly optimal design of the composite laminate for the considered loading configuration. This essentially forms the design constraints for the optimization process in terms of the required strength. The optimum design of the laminate is obtained in terms of thickness of ply, stacking sequence, number of plies at each orientation which does not fail under the considered failure criteria for a given in-plane loading configuration. In the current study, three different combinations of in-plane loadings are considered—uniaxial, biaxial and bending loads, for the multi-objective design optimization of the composite component. The results of the design optimization process i.e. the evolved optimal designs, for each loading condition are presented and discussed in the coming sections.

Further in the current work, a comparative study of the different design configurations evolved with both the fixed and variable stacking schemes has been carried out. For bending and biaxial loading configurations the design optimization has been carried by the proposed ABC model with the stacking scheme fixed and also with it being treated as a design variable. For fixing the stacking sequence, two of the most popularly used standard stacking schemes: $\{0/90\}_S$: SS_1 and $\{0/\pm45/90\}_S$: SS_2 are considered. The optimal design configurations evolved for the standard stacking schemes SS_1 and SS_2 have been compared with the optimal design configurations evolved by considering the lamina stacking sequence also as a variable of the optimization process. These comparisons shed light on the extent of influence of the lamina stacking sequence in the design optimization of the composite components.

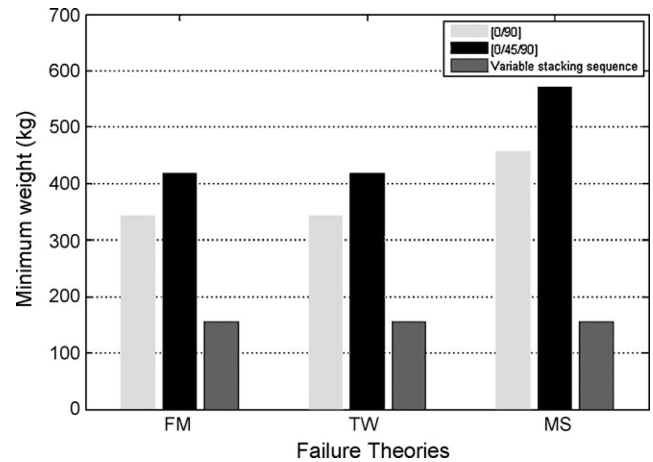

Fig. 2. Minimum weights obtained for different failure theories with standard and non-standard stacking sequences for biaxial loading condition.

6.1. Uniaxial loading

This is a fairly simple configuration of loading with the forces being applied on the component in only one direction (either in L, T or t directions). Uniaxial loading corresponds to fiber breaks and fiber compressive types of failures with uniaxial compressive loadings accounting for the fiber compressive – failures and the uniaxial tensile loads resulting in fiber breaks – failures.

We have considered a uniaxial tensile load of 1000 N/mm acting in the longitudinal direction. The results of the optimization process for this loading configuration are listed in Table 3. It can be observed from the results that the different optimal designs evolved for the three different failure criteria are quite comparable. Further it can be seen from the table that the optimal designs evolved for this loading configurations are quite simple, with a majority of the evolved design configurations resulting in a standard stacking sequence of $\{0\}$ (layers placed only at a 0° orientation) irrespective of the failure criteria considered. This is very much in-

Table 3
Representative results—optimum configurations obtained for uniaxial loading configuration.

Loading (N/mm) N_x	Failure theory	Number of Ply	Stacking sequence	Thickness (mm)	Wt (kg)	Cost ($)
		4	$[0_2]_S$	0.133	101.08	499473
		4	$[0_2]_S$	0.133	101.08	499473
	Failure mechanism based failure theory	4	$[0_2]_S$	0.133	101.08	499473
		10	$[0_5]_S$	0.053	100.7	497638
		10	$[0_5]_S$	0.053	100.7	497638
		10	$[0_5]_S$	0.053	100.7	497638
		4	$[0_2]_S$	0.133	101.08	499473
1000	Tsai-Wu failure theory	10	$[0_5]_S$	0.053	100.7	497638
		10	$[0_5]_S$	0.053	100.7	497638
		4	$[0_2]_S$	0.133	101.08	499473
		4	$[0_2]_S$	0.133	101.08	499473
		10	$[0_5]_S$	0.053	100.7	497638
	Maximum stress based failure theory	10	$[0_5]_S$	0.053	100.7	497638
		10	$[0_5]_S$	0.053	100.7	497638
		10	$[0_5]_S$	0.053	100.7	497638

Table 4
Representative results—optimum design configurations evolved for biaxial loading condition.

Loading (N/mm)			Failure theory	Number of Ply	Stacking sequence	Thickness (mm)	Wt (kg)	Cost ($)
N_x	N_y	N_{xy}						
1800	1800	60	Failure mechanism based failure theory	12	$[-15_2/75_4]_S$	0.073	166.44	822365
				16	$[-15_3/75_5]_S$	0.051	155.04	766074
				16	$[-15_3/75_5]_S$	0.051	155.04	766074
				6	$[-15_1/75_2]_S$	0.146	166.44	822365
				16	$[-15_3/75_5]_S$	0.051	155.04	766074
			Tsai-Wu failure theory	16	$[-15_3/75_5]_S$	0.051	155.04	766074
				16	$[-15_3/75_5]_S$	0.051	155.04	766074
				20	$[-15_2/-30_4/60_4]_S$	0.05	190.0	938780
				6	$[-15_1/75_2]_S$	0.146	166.44	822365
				6	$[-15_1/75_2]_S$	0.146	166.44	822365
			Maximum stress based failure theory	6	$[-15_1/75_2]_S$	0.146	166.44	822365
				6	$[-15_1/75_2]_S$	0.146	166.44	822365
				16	$[-15_3/75_5]_S$	0.051	155.04	766074
				16	$[-15_3/75_5]_S$	0.051	155.04	766074
				6	$[-15_1/75_2]_S$	0.146	166.44	822365

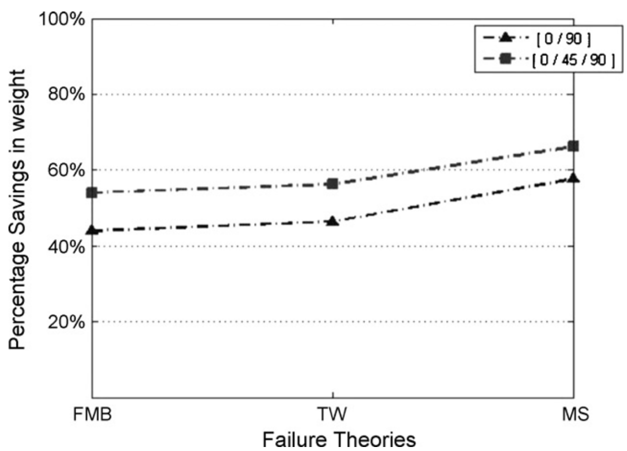

Fig. 3. Percentage weight savings for the different failure criteria between standard and non-standard stacking schemes for biaxial loading.

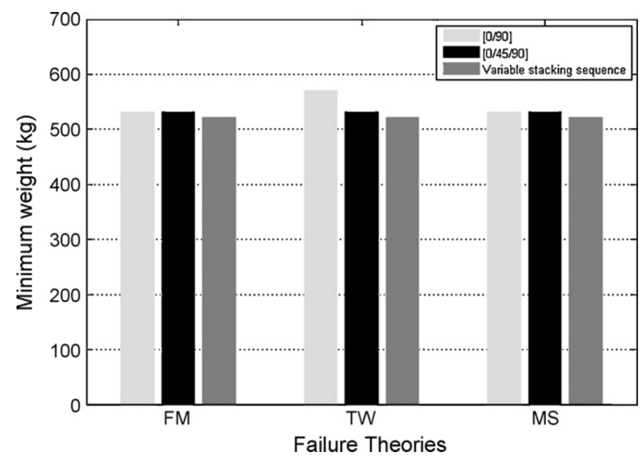

Fig. 4. Minimum weights obtained for different failure theories with standard and non-standard stacking sequences for bending loads.

line and agrees well with the strength requirements for a uniaxial loading configuration.

6.2. *Biaxial loading (with shear)*

This condition refers to loading of the composite component in two directions. We have considered a biaxial load of [1800 N/mm, 1800 N/mm] with an added shear load of 60 N/mm. The results of the multi-objective design optimization of composite laminates for this loading configuration are listed (Table 4). It can be seen that the optimum composite configurations evolved for the different failure conditions are similar.

Further, it can be observed from Table 4, that the optimal configurations suggest a number of different (non-standard) stacking sequences for the laminates. This can be attributed to the fact that the, lamina stacking scheme has also been considered as a variable for the optimization process. Stacking schemes such as $\{-15/75\}_S$, and $\{-15/-30/60\}_S$ have been evolved. These non-standard stacking schemes result in configurations with significantly lesser weights as compared to configurations with standard stacking sequences. The optimal design configurations evolved for the considered biaxial loading for the fixed stacking schemes SS_1 and SS_2 are listed in Table 5. Comparing the results listed in Tables 4 and 5 we can observe that, with the fixed stacking sequences SS_1 and SS_2, we have arrived at an optimal configuration with a minimal weight of 342 kg and 418 kg respectively. Whereas a minimum weight of 155 kg is obtained with a non-standard stacking scheme of $\{-15/75\}_S$. The comparisons of the minimum weights obtained by design optimization with the stacking sequence fixed and it being treated as a variable is given in Fig. 2. Significant savings in weight can be seen—the optimal design evolved with variable stacking sequence results in a composite

Table 5
The optimal configurations evolved for fixed standard stacking schemes of {0/90} and {0/±45/90}—for the biaxial loading condition.

Loading (N/mm)			Failure theory	Stacking sequence	Thickness (mm)	Weight (kg)
N_x	N_y	N_{xy}				
1800	1800	60	Failure mechanism based failure theory	$[0_9/90_9]_S$	0.05	342
				$[0_8/\pm45_{10}/90_4]$	0.05	418
			Tsai-Wu failure theory	$[0_9/90_9]_S$	0.05	342
				$[0_{10}/\pm45_6/90_6]$	0.05	418
			Maximum stress based failure theory	$[0_{12}/90_{12}]_S$	0.05	456
				$[0_8/\pm45_{18}/90_4]$	0.05	570

Table 6
Representative results—optimum design configurations evolved for bending loads.

Loading (N/mm) M_x	Failure theory	Number of Ply	Stacking sequence	Thickness (mm)	Wt (kg)	Cost ($)
		22	$[0_{10}/30_1]_S$	0.125	522.50	2581414
		22	$[0_9/45_2]_S$	0.125	522.50	2581414
	Failure mechanism based failure theory	14	$[0_7]_S$	0.196	521.36	2575746
		14	$[0_7]_S$	0.196	521.36	2575746
		14	$[0_7]_S$	0.196	521.36	2575746
		22	$[0_9/45_2]_S$	0.125	522.50	2581414
		14	$[0_7]_S$	0.196	521.36	2575746
−1800	Tsai-Wu failure theory	14	$[0_7]_S$	0.196	521.36	2575746
		14	$[0_7]_S$	0.196	521.36	2575746
		14	$[0_7]_S$	0.196	521.36	2575746
		14	$[0_7]_S$	0.196	521.36	2575746
		14	$[0_7]_S$	0.196	521.36	2575746
	Maximum stress based failure theory	22	$[0_{10}/30_1]_S$	0.125	522.50	2581414
		14	$[0_7]_S$	0.196	521.36	2575746
		14	$[0_7]_S$	0.196	521.36	2575746

Table 7
The optimal configurations evolved for fixed standard stacking schemes of {0/90} and {0/±45/90}—for bending loads.

Loading (N/mm) M_x	Failure theory	Stacking sequence	Thickness (mm)	Weight (kg)
	Failure mechanism based failure theory	$[0_{13}/90_1]_S$	0.1	532
		$[0_{21}/\pm45_3/90_1]$	0.05	532
−1800	Tsai-Wu failure theory	$[0_{14}/90_1]_S$	0.1	570
		$[0_{23}/\pm45_1/90_3]$	0.05	532
	Maximum stress based failure theory	$[0_{13}/90_1]_S$	0.1	532
		$[0_{21}/\pm45_3/90_1]$	0.05	532

component with nearly half the weight of components with fixed stacking schemes. The percentage weight savings with variable stacking sequence as compared to standard stacking sequences SS_1 and SS_2 are given in Fig. 3. This reaffirms the fact that the stacking scheme is a crucial parameter for the design optimization of composite laminates. Hence, including this as a design variable in the optimization process results in an optimal solution.

6.3. Bending loads

Further, we have also considered a case with bending loads for the design optimization of the composite component. The results obtained for this loading configuration are listed in Table 6. Again, as in the previous case it can be seen form Table 6, that a number of different optimal design configurations with non-standard stacking sequences have been evolved by the ABC model. Table 7 lists the optimal design configurations evolved for the fixed stacking schemes SS_1 and SS_2. Fig. 4 depicts the minimum weights obtained with fixed standard stacking schemes and variable stacking scheme. For both the standard schemes of SS_1 and SS_2 optimal design configurations with minimum weight of 532 kg respectively, for all the three failure criteria are evolved. The optimal design configuration evolved by ABC model, with the variable stacking sequence has a minimal weight of around 521 kg. The difference in weights is a mere 3% whereas with biaxial loading weight savings is up to 50%. This can be attributed to a possibility that the stacking sequence does not have a pronounced

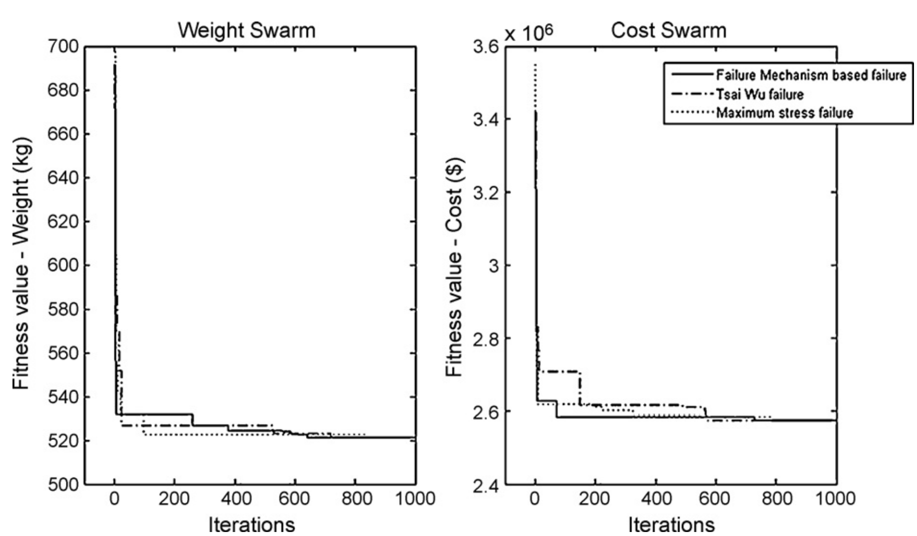

Fig. 5. Convergence patterns of both the weight and cost swarm for the different failure criteria.

effect on the bending stresses developed in a laminated composite.

Here, it can be observed that the objectives more or less reinforce each other rather than being completely conflicting, both tending towards a configuration with lower weight. Both the objectives—weight and costs are directly proportional to the number of plies. But the manufacturing cost increases with the increase in the number of layers and also with reduction in ply thickness. Finally, a configuration of lower ply and higher thickness is obtained as optimal. This trend can be observed in all the test cases and for all the three failure criteria considered.

The convergence pattern of the swarm population for the different failure criteria for a given loading configuration is illustrated in Fig. 5. In all the cases considered, the artificial bees converge and the end condition is satisfied at 1000 iterations.

7. Comparison of nature inspired techniques

Nature inspired techniques provide a more robust and efficient approach for solving complex real-world problems [34,35]. In recent years, many nature inspired techniques such as Genetic Algorithm (GA) [36], Artificial Immune System (AIS) [13], Particle Swarm Optimization (PSO) [11], etc have been applied to optimal structural design.

Nagendra et al. [37] applied GA for the design for minimum weight stiffened panels with buckling constraints. The study showed that the GA discrete design procedure was superior to other alternatives for both stiffened panels with and without cutouts. Malott et al. [38] carried out the optimal design of laminated composite sandwich panels with bending–twisting coupling using GA. Narayana Naik et al. [39] applied GA for the design optimization of composites using single-objective optimization for weight based on the same failure criterion as used in this paper. Here the effect of stacking sequence and loading on the optimum weight of the laminate is studied and it is found that the optimum weight of the laminate varies with the stacking sequence of the laminate. Ramoǐn et al. [40] implemented a micro-genetic algorithm to carry out the multi-objective optimization of the drilling process of a laminate composite material. Jacob and Senthil [41] used a multi-objective GA for optimization of composites for strength, stiffness and minimal mass.

In the current work we employ the principle of vector evaluated Genetic Algorithm [30] for solving the given multi-objective optimization problem. In VEGA the population is divided into m different parts for n different objectives; the selection operation is done for each objective separately, filling equal portions of mating pool [30]. Afterwards, the mating pool is shuffled, and crossover and mutation are performed as usual.

Omkar et al. [11,13] have used the swarm intelligence and immuno-computing techniques which includes PSO [11] and AIS [13] respectively for the design optimization of laminate composite structures. The design variables, constraints and failure criteria that are used are same as in this paper. A comparison of the results obtained by these three methods and ABC is shown in Table 8. The results of the optimization process for the uniaxial and bending loading configuration are listed in Table 8. It can be observed from the results that the different optimal designs evolved for the three different failure criteria are quite comparable in all the nature inspired techniques. In the case of biaxial loading configuration, the results of the multi-objective design optimization of composite laminates are better in ABC in comparison with that of AIS and GA. Therefore the performance of ABC is on par with that of PSO, AIS and GA for all the loading configurations.

Table 8
Comparison of results obtained by PSO, AIS, GA and ABC algorithms.

Failure theory	Particle Swarm Optimization (PSO)		Artificial Immune System (AIS)		Genetic Algorithm (GA)		Artificial Bee Colony (ABC)	
	Minimum weight (kg)	Minimum cost (×1000) ($)	Minimum weight (kg)	Minimum cost (×1000) ($)	Minimum weight (kg)	Minimum cost (×1000) ($)	Minimum weight (kg)	Minimum cost (×1000) ($)
Uniaxial loading configuration								
Failure mechanism based failure theory	100.52	496.7	100.7	497.6	100.7	497.6	100.7	497.6
Tsai-Wu failure theory	100.52	496.7	100.7	497.6	100.7	497.6	100.7	497.6
Maximum stress based failure theory	100.52	496.7	100.7	497.6	100.7	497.6	100.7	497.6
Biaxial loading configuration								
Failure mechanism based failure theory	166	821.0	190	938.7	158.08	781.1	155.04	766.1
Tsai-Wu failure theory	165	823.7	190	938.7	166.44	822.3	155.04	766.1
Maximum stress based failure theory	156	820.6	190	938.7	158.08	781.1	155.04	766.1
Bending loading configuration								
Failure mechanism based failure theory	521	2575.5	523.26	2585.2	523.26	2585.2	521.36	2575.7
Tsai-Wu failure theory	521.3	2575.7	524.40	2590.9	523.26	2585.2	521.36	2575.7
Maximum stress based failure theory	518.7	2562.6	523.26	2585.2	522.88	2583.3	521.36	2575.7

8. Conclusions

In this paper, we present a generic model for composite design optimization based on VEABC. In the current work, based on the ABC algorithm a new version of VEABC algorithm for discrete variables has been developed and implemented successfully for the multi-objective design optimization of composites. The composite design optimization problem has been formulated as multi-objective optimization problem with objectives of minimizing weight of the component for a required strength and minimizing the total cost incurred. The number of layers, layer thicknesses and the stacking sequence—have been considered as the design variables for design optimization. The composite design is evaluated based on several failure criteria such as Maximum stress, Tsai-Wu and Failure mechanism based failure criteria. The proposed optimization model has been validated for a number of different in-plane loading configurations from different regions of the failure envelope. Also, a comparison has been brought out between configuration evolved, with the stacking schemes fixed and the stacking scheme being treated as a variable of the optimization process. These comparisons bring out the significant weight savings obtained by treating the stacking sequence also as a design variable instead of fixing it any of the commonly used standard stacking schemes like $\{0/90\}_S$ and $\{0/\pm45/90\}_S$. It can be seen from the results that, in all the considered cases the VEABC based optimization model has performed quite satisfactorily in comparison with other nature inspired techniques, evolving superior composite design that results in significant weight savings. This comprehensively ascertains the robustness of the proposed method. Further, this approach does not impose any limitation on the number of objectives and constraints. The VEABC based optimization model developed in this paper allows for easy incorporation of any changes in design parameters. Inclusion of further constraints and objectives can be effected without necessitating any major changes to current framework, making this VEABC based optimization model robust and generic, providing great deal of flexibility to extend it to any number of different composite configurations.

Acknowledgment

This work is supported by Aeronautical Research and Development Board, Defence Research and Development Organisation, New Delhi, India.

References

[1] Z. Gürdal, R.T. Haftka, P. Hajela, Design and Optimization of Laminated Composite Materials, Wiley-Interscience, 1999.
[2] Z. Gürdal, R.T. Haftka, Optimization of composite laminates, Presented at the NATO Advanced Study Institute on Optimization of Large Structural Systems, Germany, 1991.
[3] M. Bruyneel, A general and effective approach for the optimal design of fibre reinforced composite structures, Composites Science and Technology 66 (2006) 1303–1314.
[4] D.K. Shin, Z. Gürdal, O.H. Griffin, Minimum weight design of laminated plates for post-buckling performance, in: Proceedings of the 32nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, 1991.
[5] S. Adali, M. Walker, V.E. Verijenko, Multi-objective optimisation of laminated plates for maximum pre-buckling, buckling and post-buckling strength using continuous and discrete ply angles, Composite Structures 35 (1996) 117–130.
[6] N. Kumar, T.R. Tauchert, Multi-objective design of symmetrically laminated plates, Transactions of the ASME 114 (1992) 620–625.
[7] J.L. Pelletier, S. Senthil Vel., Multi-objective optimization of fibre reinforced composite laminates for strength, stiffness and minimal mass, Computers and Structures 1016 (2006).
[8] S. Venkataraman, R.T. Haftka, Optimization of composite panels—a review, in: Proceedings of the 14th Annual Technical Conference of the American Society of Composites, Dayton, OH, 1999, pp. 479–488.
[9] R. Le Riche, R.T. Haftka, Optimization of laminate stacking sequence for buckling load maximization by genetic algorithm, AIAA Journal 31 (5) (1993).
[10] A.K. Garg, D. Roy Mahapatra, S. Suresh, S. Gopalakrishna, S.N. Omkar, Estimation of composite design model parameters using spectral element and neural networks, Composite Science and Technology 64 (16) (2007) 2477–2494.
[11] S.N. Omkar, D. Mudigere, G. Narayana Naik, S. Gopalakrishna, Vector evaluated particle swarm optimization (VEPSO) for multi-objective design optimization of composite structures, Computers & Structures 86 (2008) 1–14.
[12] K.E. Parsopoulos, D.E. Tasoulis, M.N. Vrahatis, Multi-objective optimization using parallel vector evaluated particle swarm optimization, in: Proceedings of International Conference on Artificial Intelligence and Applications (IASTED), 2004.
[13] S.N. Omkar, R. Khandelwal, S. Yathindra, G. Narayana Naik, S. Gopalakrishnan, Artificial immune system for multi-objective design optimization of composite structures, Engineering Applications of Artificial Intelligence 21–8 (2008) 1416–1429.
[14] G. Soremekun, Z. Gürdal, R.T. Haftka, L.T. Watson, Composite laminate design optimization by genetic algorithm with generalized elitist selection, Computers & Structures 79–2 (2001) 131–143.
[15] I. Rajendran, S. Vijayarangan, Optimal design of a composite leaf spring using genetic algorithms, Computers & Structures 79 (2001) 1121–1219.
[16] J.H. Park, J.H. Hwang, C.S. Lee, W. Hwang, Stacking sequence design of composite laminates for maximum strength using genetic algorithms, Composite Structures 52 (2001) 217–231.
[17] L. Grosset, S. Venkataraman, R.T. Haftka, Genetic optimization of two-material composite laminates, in: Sixteenth Technical Conference of the American Society for Composites, USA, 2001.
[18] M. Walker, R.E. Smith, A technique for the multi-objective optimisation of laminated composite structures using genetic algorithms and finite element analysis, Composite Structures 62 (2003) 123–128.
[19] L. Costa, L. Fernandes, I. Figueiredo, J. Juǐ dice, R. Leal, P. Oliveira, Multiple- and single-objective approaches to laminate optimization with genetic algorithms, Structural Multidisciplinary Optimization 27 (2004) 55–65.
[20] D. Karaboga, B. Basturk, On the performance of Artificial Bee Colony (ABC) algorithm, Applied Soft Computing 8 (1) (2008) 687–697.
[21] X.S. Yang, Engineering Optimizations via Nature-Inspired Virtual Bee Algorithms, Lecture Notes in Computer Science, vol. 3562, Springer–Verlag GmbH, 2005, pp. 317–323.
[22] A.A. Groenwold, R.T. Haftka, Optimization with non-homogeneous failure criteria like Tsai-Wu for composite laminates, Structural Multidisciplinary Optimization 32 (2006) 183–190.
[23] G. Narayana Naik, A.V. Krishna Murty, S. Gopalakrishnan, A failure mechanism based failure theory for laminated composites including the effect of shear stress, Composite Structures 69–2 (2005) 219–227.
[24] K. Deb, Multi-objective Optimization Using Evolutionary Algorithms, John Wiley and Sons Ltd., 2001.
[25] V. Pareto, Manual of Political Economy, A.M. Kelley, New York, 1971.
[26] K.V. Frisch, Bees: Their Vision, Chemical Senses and Language, Cornell University Press, Ithaca, NY, 1976 (Revised edition).
[27] K. Von Frisch, The Dance Language and Orientation of Honey Bees, Harvard University Press, 1967.
[28] D.T. Pham, A. Ghanbarzadeh, Multi-Objective Optimization Using the Bees Algorithm, in: Proceedings of IPROMS 2007 Conference.
[29] D. Teodorovic, P. Lucic, G. Markovic, M. Dell Orco, Bee colony optimization: principles and Applications, Neural Network Applications in Electrical Engineering (2006) 151–156.
[30] J.D. Schaffer, Multi-objective optimization with vector evaluated genetic algorithms, Ph.D. Thesis, Vanderbilt University, Nashville, USA, 1984.
[31] C.A. Coello Coello, M.R. Sierra, A Study of the Parallelization of a Co-evolutionary Multi-objective Evolutionary Algorithm. Lecture Notes in Artificial Intelligence, vol. 2972, Springer–Verlag, 2004, pp. 688–697.
[32] G. Kovacs, A.A. Groenwold, K. Jǐarmai, J. Farkas, Analysis and optimum design of fibre-reinforced composite structures, Structural Multidisciplinary Optimization 28 (2004) 170–179.
[33] R.C. Eberhart, X. Hu, Human tremor analysis using particle swarm optimization, in: Proceedings of the IEEE Congress on Evolutionary Computation (CEC), 1999, pp. 1927–1930.
[34] T. Bäck, H.P. Schwefel, An overview of evolutionary algorithms for parameter optimization, Evolutionary Computation 1 (1) (1993).
[35] X. Yao (Ed.), Evolutionary Computation: Theory and Applications, World Scientific, Singapore, 1999.
[36] D.E. Goldberg, Genetic Algorithms in Search, Optimization, and Machine Learning, Pearson Education, Inc., India, 2005.
[37] S. Nagendra, R.T. Haftka, Z. Gurdal, Design of a blade stiffened composite panel by a genetic algorithm, AIAA/ASME Adaptive Structures Forum, Technical Papers, Pt. 4(A93-3387613-39), La Jolla, CA, 1993, pp. 2418–2436.
[38] B. Malott, R.C. Averill, E.D. Goodman, Y. Ding, W.F. Punch, Use of genetic algorithms for optimal design of laminated composite sandwich panels with bending–twisting coupling, AIAA Journal 37 (1996) 1874–1881.
[39] G. Narayana Naik, S. Gopalakrishnan, R. Ganguli, Design optimization of composites using genetic algorithms and failure mechanism based failure criterion, Composite Structures 83–4 (2008) 354–367.
[40] Q.S. Ramoǐn, R. Pedro, P.J. Davim, Multi-objective optimization of cutting parameters for drilling laminate composite materials by using genetic algorithms, Composites Science and Technology 66 (15) (2006) 3083–3088.
[41] L.P. Jacob, S.V. Senthil, Multi-objective optimization of fiber reinforced composite laminates for strength, stiffness and minimal mass, Computers & Structures 84 (29–30) (2006) 2065–2080.