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Portfolio optimization problems in different risk measures using genetic algorithm

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ABSTRACT

This paper introduces a heuristic approach to portfolio optimization problems in different risk measures by employing genetic algorithm (GA) and compares its performance to mean–variance model in cardinality constrained efficient frontier. To achieve this objective, we collected three different risk measures based upon mean–variance by Markowitz; semi-variance, mean absolute deviation and variance with skewness. We show that these portfolio optimization problems can now be solved by genetic algorithm if mean–variance, semi-variance, mean absolute deviation and variance with skewness are used as the measures of risk. The robustness of our heuristic method is verified by three data sets collected from main financial markets. The empirical results also show that the investors should include only one third of total assets into the portfolio which outperforms than those contained more assets.

1. Introduction

Expect return and risk are the most important parameters with regard to portfolio optimization problems. One of the main contributions on this problem is by Markowitz (1952, 1991) who introduced mean–variance model, but the standard mean–variance model is based on assumption that investors are risk averse and the return of assets are normally distributed. Jia and Dyer (1996) noted that these conditions are rarely satisfied in practice. The mean–variance objective function may not be the best choice available to investors in terms of an appropriate risk measure. Furthermore, other risk measures may be more appropriate. From a practical point of view, real world investors have to face a lot of constraints in risk models: trading limitation, size of portfolio, etc. Such as constraints may be formed in a nonlinear mixed integer programming problem which is considerably more difficult to solve than the original model. Several researchers have attempted to find this problem by a variety of techniques, but exact solution methods fail to solve large-scale instances of the problem. Therefore, several researchers try to improve algorithms by using the state-of-the-art mathematical programming methodology to solving portfolio problems. The purpose of this paper is to show that portfolio optimization problems containing cardinality constrained efficient frontier can be successfully solved by the state-of-the-art genetic algorithms if we use the different risk measures such as mean–variance, semi-variance, mean absolute deviation and variance with skewness. We also show that practical portfolio optimization problems consisting of different numbers of assets drawn from three main markets stock indices can be solved by a genetic algorithm within a practical amount of time.

The remainder of this paper is organized as follows. Section 2 describes the portfolio optimization in the risk measures which we want to solve. In Section 3 investigates basic structure of genetic algorithm. Section 4, our proposed algorithm was introduced. Section 5 provides our computational results using C++ programming. It shows that cardinality constrained portfolio optimization problems can be solved in different risk measures without difficulty. Conclusion is given in Section 6.

2. Portfolio optimization in the risk measures

Portfolio is to deal with the problem of how to allocate wealth among several assets. The portfolio optimization problems have been one of the important research fields in modern risk management. In generally, an investor always prefers to have the return on their portfolio as large as possible. At the same time, he also wants to make the risk as small as possible. However, a high return always accompanied with a higher risk. Markowitz introduced the mean–variance model, which has been regarded as a quadratic programming problem. In spite of its popularity during the past, the mean–variance model is based upon the assumptions that an investor is risk averse and that either (i) the distribution of the rate of return is multivariate normal or (ii) the utility of the investor is a quadratic function of the rate of return. Unfortunately however, neither (i) nor (ii) holds in practice. It is now widely recognized that the real world portfolios do not follow a multivariate normal distribution. Many researchers once suggest that cannot blindly

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depend on mean–variance model. Therefore, there has been a tremendous amount of researches on improving this basic model both computationally and theoretically. Various risk measures such as semi-variance model, mean absolute deviation model and variance with skewness model have been proposed. Among them risk models were mathematically shown as below.

2.1. Mean–variance model

Markowitz was the first to apply variance or standard deviation as a measure of risk. He assumed that his classical formation is as follows:

$$\text{Minimize} \quad \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{1}$$

$$\text{Subject to} \quad \sum_{i=1}^{N} w_i \mu_i = R^* \tag{2}$$

$$\sum_{i=1}^{N} w_i = 1 \tag{3}$$

$$0 \leqslant w_i \leqslant 1, \quad i = 1,\ldots,N \tag{4}$$

where

N is the number of assets available;
w_i is the proportion $(0 \leqslant w_i \leqslant 1)$ of the portfolio held in assets i $(i = 1,\ldots,N)$;
μ_i is the expected return of asset i $(i = 1,\ldots,N)$;
σ_{ij} is the covariance between assets i and j $(i = 1,\ldots,N;$ $j = 1,\ldots,N)$.

Eq. (1) minimizes the total variance (risk) associated with the portfolio while Eq. (2) ensures that the portfolio has an expected return of R^*. Eq. (3) ensures that the proportions add to one. In Eq. (4) the proportion held in each asset is between zero (minimum amount) and one (maximum amount). This formulation (Eqs. (1)–(4)) is a quadratic programming problem and nowadays it can be solved optimally using available software tool.

By solving the above optimization problem continuously with a different R^* each time, a set of efficient points is traced out. This efficient set called the efficient frontier and is a curve that lies between the global minimum risk portfolio and the maximum return portfolio. In other words, the portfolio selection problem is to find all the efficient portfolios along this frontier.

In order to enrich the model, we introduce a weighting parameter λ $(0 \leqslant \lambda \leqslant 1)$ and consider:

$$\text{Minimize} \quad \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} w_i \mu_i\right] \tag{5}$$

$$\text{Subject to} \quad \sum_{i=1}^{N} w_i = 1 \tag{6}$$

$$\sum_{i=1}^{N} z_i = K \tag{7}$$

$$\varepsilon_i z_i \leqslant w_i \leqslant \delta_i z_i, \quad i = 1,\ldots,N \tag{8}$$

$$z_i \in \{0,1\}, \quad i = 1,\ldots,N \tag{9}$$

where

K is the desired number of assets in the portfolio;
ε_i is the minimum proportion that must be held of asset i $(i = 1,\ldots,N)$ if any of assets i is held;
δ_i is the maximum proportion that can be held of asset i $(i = 1,\ldots,N)$ if any of assets i is held;
$z_i = 1$ if any of asset i $(i = 1,\ldots,N)$ is held
$= 0$ otherwise.

Eq. (5) the case $\lambda = 0$ represents maximum expected return and $\lambda = 1$ represents minimum risk. Values of λ satisfying $0 < \lambda < 1$ represent an explicit trade-off between risk and return, generating solutions between the two extremes $\lambda = 0$ and $\lambda = 1$. Eq. (6) ensures that the proportions add to one. Eq. (7) is assets desired number constraint. It ensures that exactly K assets are held. Eq. (8) constraints define lower and upper limits on the proportion of each asset which can be held in the portfolio. It ensures that if any of assets i is held $(z_i = 1)$ its proportion w_i must lie between ε_i and δ_i, while if none of asset i is held $(z_i = 0)$ its proportion w_i is zero. Eq. (9) is the integrality constraint. By a weighting parameter λ, we could use this program (Eqs. (5)–(9)) to trace out the cardinality constrained efficient frontier (CCEF) in an exactly analogous way. The use of heuristics for cardinality constrained portfolio optimization has been proposed and discussed by Chang, Meade, Beasley, and Sharaiha (2000).

2.2. Semi-variance model

Standard mean–variance model is based upon assumptions that an investor is risk averse and that the distribution of the rate of return is multivariate normal. This means that the variance component of the Markowitz quadratic objective function can be replaced by other risk functions such as semi-variance. With an asymmetric return distribution, the mean–variance approach leads to an unsatisfactory prediction of portfolio behavior. Markowitz indeed suggested that a model based on semi-variance would be preferable. Let:

T be such that we have observed historical values for stocks over the time period $0,1,2,\ldots,T$;
v_{it} be the value of one unit of stock i $(i = 1,\ldots,N)$ at time t $(t = 0,\ldots,T)$;
C_{cash} be the cash available to invest in the portfolio;
x_i be the number of units of stock i $(i = 1,\ldots,N)$ that we choose to hold in the portfolio;
$z_i = 1$ if any of stock i $(i = 1,\ldots,N)$ is held in the portfolio
$= 0$ otherwise.

It is helpful when formulating the problem to introduce:

w_i is the proportion of C_{cash} that is invested at time T in stock i $(i = 1,\ldots,N)$;
r_t is the single period continuous time return given by the portfolio at time t $(t = 1,\ldots,T)$.

We get the values through the variables given previously:

$$w_i = v_{iT} x_i / C_{cash}, \quad i = 1,\ldots,N \tag{10}$$

$$r_t = \log_e\left(\sum_{i=1}^{N} v_{it} x_i \Big/ \sum_{i=1}^{N} v_{it-1} x_i\right), \quad t = 1,\ldots,T \tag{11}$$

Eq. (10) defines w_i to be the proportion of the portfolio associated with stock i at time T and Eq. (11) defines r_t to be the return on the portfolio (since the total value of the portfolio at time t is $\sum_{i=1}^{N} v_{it} x_i$). Then the constraints associated with discrete time portfolio optimization problem are

$$\sum_{i=1}^{N} z_i = K \tag{12}$$

$$\varepsilon_i z_i \leqslant v_{iT} x_i / C_{cash} \leqslant \delta_i z_i, \quad i = 1,\ldots,N \tag{13}$$

$$\sum_{i=1}^{N} v_{iT} x_i = C_{cash} \tag{14}$$

$$x_i \geqslant 0, \quad i = 1,\ldots,N \tag{15}$$

$$z_i \in \{0,1\}, \quad i = 1,\ldots,N \tag{16}$$

Algebraically we can transform the above equations to form constraints in decision variables $[w_i]$ and $[z_i]$, this algebraic transformation yields the constraints

$$\sum_{i=1}^{N} z_i = K \tag{17}$$

$$\varepsilon_i z_i \leqslant w_i \leqslant \delta_i z_i, \quad i = 1, \ldots, N \tag{18}$$

$$\sum_{i=1}^{N} w_i = 1 \tag{19}$$

$$r_t = \log_e\left[\left(\sum_{i=1}^{N} w_i v_{it}/v_{iT}\right) \Big/ \left(\sum_{i=1}^{N} w_i v_{it-1}/v_{iT}\right)\right], \quad t = 1, \ldots, T \tag{20}$$

$$w_i \geqslant 0, \quad i = 1, \ldots, N \tag{21}$$

$$z_i \in \{0, 1\}, \quad i = 1, \ldots, N \tag{22}$$

Furthermore, it is helpful at this stage to define:

$$\bar{r} = \sum_{t=1}^{T} r_t/T \tag{23}$$

so that $\bar{r}$ is the mean portfolio return over the time period $1, 2, \ldots, T$. Therefore, semi-variance only considers downside risk (returns below $\bar{r}$) and is defined by

$$\sum_{t=1;\ r_t<\bar{r}}^{T} (r_t - \bar{r})^2/T \tag{24}$$

In other words risk is no longer symmetric, and we are not concerned with time periods in which $r_t \geqslant \bar{r}$ (portfolio return exceeds mean return). Further discussion of downside risk can be found in Sortino and Forsey (1996) and Sortino and van der Meer (1991).

Hence the semi-variance model for the CCEF is:

$$\text{Minimize} \quad \lambda\left[\sum_{t=1;\ r_t<\bar{r}}^{T} (r_t - \bar{r})^2/T\right] - (1-\lambda)\bar{r} \tag{25}$$

Subject to Eqs. (17)–(23)

Many researchers have studied the semi-variance model. Konno, Waki, and Yuuki (2002) showed that very large-scale mean-lower partial risk models are solvable in an efficient manner by the mathematical programming methodology. In the large-scale stock portfolio, they observe virtually no difference in the tail distribution among portfolios generated by mean–variance model and mean–lower semi-variance model above 60 monthly data of 1100 stocks on the Tokyo Stock Exchange. Enrique (2005) proposed mean–semi-variance approach with those derived from the traditional mean–variance model. Computational results were presented among the seven scenarios (seven different target expect returns), four of them have the same solutions for both measures while three of them give different solutions.

2.3. Mean absolute deviation model

Konno and Yamazaki (1991) first propose a mean absolute deviation portfolio optimization model as an alternative to the Markowitz mean–variance portfolio selection model, which allows the portfolio selection problem to be formulated and solved via linear programming. In their paper computational results were presented for one problem involving 224 assets. They showed that using mean absolute deviation model generates similar results to the Markowitz mean–variance model. The advantage of the mean absolute deviation model is linear programming problem instead of quadratic programming problem, and is easily solved by means of classical techniques for large-scale optimization problems. Our method has the flexibility to incorporate a mean absolute deviation function, but they may not have any computational advantages over classical approaches. The mean absolute deviation model for the CCEF is:

$$\text{Minimize} \quad \lambda\left[\sum_{t=1}^{T} |r_t - \bar{r}|/T\right] - (1-\lambda)\bar{r} \tag{26}$$

Subject to Eqs. (17)–(23)

The computational advantages of the mean absolute deviation model over the mean–variance are also once well established by Konno (2003) and Konno and Koshizuka (2005).

2.4. Variance with skewness

The importance of the third order moment in portfolio optimization was first suggested by Samuelson (1958). A portfolio return may not be a symmetric distribution. The distribution of individual asset returns tends to exhibit a higher probability of extreme values than is consistent with normality.

In order to capture the characteristics of the return distribution and provide further decision-making information to investors, another approach is to include skewness into the mean–variance model. Hence the variance with skewness model for the CCEF is:

$$\text{Minimize} \quad \lambda\left[\sum_{t=1}^{T} (r_t - \bar{r})^2/T\right] - (1-\lambda)\bar{r} - \theta\left[\left(\sum_{t=1}^{T} (r_t - \bar{r})^3/T\right) \Big/ \left(\sum_{t=1}^{T} (r_t - \bar{r})^2/T\right)^{3/2}\right] \tag{27}$$

Subject to Eqs. (17)–(23)

where θ is a weighing factor for skewness and the expression it weights in Eq. (27) is known as the coefficient of skewness. Eq. (27) balances variance, expected return and skewness at the same time. For a specific value of θ, we can thus generate an efficient frontier which reflects our attitude to skewness. Although positive skewness in portfolio returns implies some reduction in downside risk, which is favorable to investors, the coefficient of skewness is affected by returns greater than, as well as those less than, the mean return. Therefore, it is possible to have portfolios with similar skewness but quite different downside behavior.

Although many researchers have demonstrated the existence of skewness in portfolios, only a few studies to date have proposed incorporating skewness into the portfolio optimization problem. In recent year, Lai (1991) proposed a polynomial goal programming algorithm to solve the unconstrained portfolio optimization problem with skewness. Konno and Suzuki (1995) have considered a mean–variance objective function extended to include skewness and applied piecewise linear approximation to the objective function. Computational results were provided for three data sets involving 225 assets. Konno and Yamamoto (2005) also show that a mean–variance skewness portfolio optimization model can be solved exactly and fast by using the integer programming approach. Note here that, as far as we are aware, there have been no applications of the variance with skewness objective function to the cardinality constrained portfolio optimization problem reported in the literature.

In the next section, we will review the basic principle and terminology of genetic algorithms and they were applied in different risk measures.

3. Genetic algorithms

Based on the Darwin principle "the fittest survive" in nature, genetic algorithm (GA) was first initiated by Holland's (1975) and has rapidly become the best-known evolutionary techniques (Goldberg, 1997; Mitchell, 1996). Since the pioneering method by Holland, numerous related GA-based portfolio selection approaches have been published. Arnone, Loraschi, and Tettamanzi (1993) presented a GA for the unconstrained portfolio optimization problem with the risk associated with the portfolio being measured by downside risk. Kyong, Tae, and Sungky (2005) also used GA to support portfolio optimization for index fund management. Lin and Liu (2008) proposed that GA for portfolio selection problems with minimum transaction lots. Recently, GA has attracted much attention in portfolio optimization problems.

In GA, an initial population containing constant number of chromosomes is generated randomly. With regard to portfolio optimization problems, each chromosome represents the weight of individual stock of portfolio and is optimized to reach a possible solution. An evaluation function is formed to evaluate the fitness for each chromosome, which defines how good a solution the chromosome represents. By using crossover, mutation values and natural selection, the population will converge to one containing only chromosomes with good fitness. Where the larger the fitness value is, the better objective function value the solution has. The basic steps in GA are shown as follows:

Step 1: Initialize a randomly generated population.
Step 2: Evaluate fitness of individual in the population.
Step 3: Apply elitist selection: carry on the best individuals to the next generation from reproduction, crossover, and mutation.
Step 4: Replace the current population by the new population.
Step 5: If the termination condition is satisfied then stop, else go to Step 2.

Through this reproduction once, the children of two chromosomes are generated. The reproduction process is operated until all chromosomes of a new population have been generated thoroughly. Through specified maximum generations, the best solution ever found is the answer.

4. The proposed GA portfolio optimization

The proposed genetic algorithm for portfolio optimization problems based on the GA steps discussed in the previous section. This section we will describe in detail how to implement the proposed method.

4.1. Population initialization

This paper used a population size of 100. Parents were chosen by binary tournament selection which works by forming two pools of individuals, each consisting of two individuals drawn from the population randomly. The individuals with the best fitness, one taken from each of the two tournament pools, are chosen to be parents.

4.2. Fitness objective function evaluation

We used mean–variance objective function

$$f = \lambda\left[\sum_{i\in Q}\sum_{j\in Q} w_i w_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i\in Q} w_i \mu_i\right]$$

as an example to show a feasible solution in the portfolio optimization problem. The chromosome representation of a solution has two distinct parts, a set Q of K distinct assets and K real numbers s_i $(0 \leqslant s_i \leqslant 1)$, $i \in Q$. Now given a set Q of K assets a fraction $\sum_{j\in Q}\varepsilon_j$ of the total portfolio is already accounted for and so we interpret s_i as relating to the share of the free portfolio proportion $1-\sum_{j\in Q}\varepsilon_j$ associated with asset $i \in Q$. Hence the proportion associated with asset i in the portfolio is given by

$$w_i = \varepsilon_i + \left(s_i \Big/ \sum_{j\in Q} s_j\right)\left(1-\sum_{j\in Q}\varepsilon_j\right)$$

i.e. the minimum proportion plus the appropriate share of the free portfolio proportion.

Not all possible chromosomes correspond to feasible solutions (because of the constraint (Eq. (8)) relating to the limits on the proportion of an asset that can be held). In GA evaluation we can automatically ensure that the constraints relating to the lower limits ε_i are satisfied in a single algorithmic step. However we need an iterative procedure to ensure that the constraints relating to the upper limits δ_i are satisfied.

4.3. Reproduction, crossover, and mutation

In this section we will describe how the genetic operators are modified and how they performed in our algorithm. Children in our GA are generated by uniform crossover. In uniform crossover two parents have a single child. If an asset i is present in both parents it is present in the child (with an associated value s_i randomly chosen from one or other parent). If an asset i is present in just one parent it has probability 0.5 of being present in the child. Children are also subject to mutation, multiplying by 0.9 or 1.1 (chosen with equal probability) the value $(\varepsilon_i + s_i)$ of a randomly selected asset i. This mutation corresponds to decreasing or increasing this value by 10%.

4.4. Replacement

We used a steady-state population replacement strategy. With this strategy each new child is placed in the population as soon as it is generated. We choose to replace the member of the population with the worst objective function value.

4.5. Termination criterion

With regard to all the computational results reported in this paper we examined 500 different λ values. With regard to the number of iterations we used 1000N for our GA heuristic. These values mean that the heuristic evaluates exactly 1000N solutions for each value of λ.

5. Research results

In the section, we report the computational results on different risk measures under cardinality constraints. We used historical daily data collected in the HANG SENG, FTSE and S&P 100 with price data of 33, 93 and 99 assets respectively from January 2004 to December 2006. The cardinality constraint K is set from 10 to 90 increased by 10 each time. We solved the problem using heuristic genetic algorithm is coded in C++ and run on a personal computer.

The comparisons of the proposed portfolio optimization for different number of assets (K = 10 to 90) are presented from Figs. 1–6.

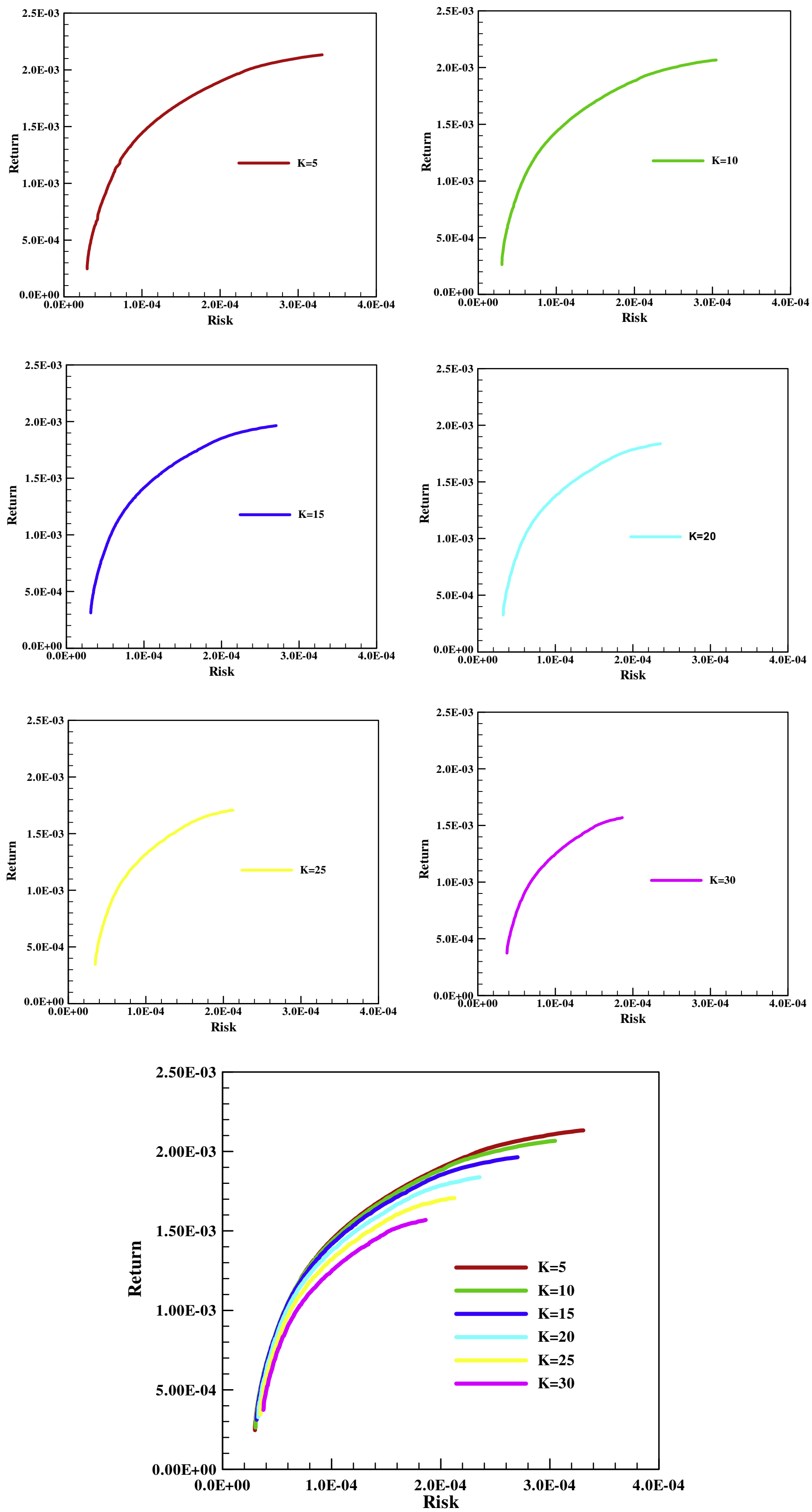

Fig. 1. CCEF of mean–variance model for HS data.

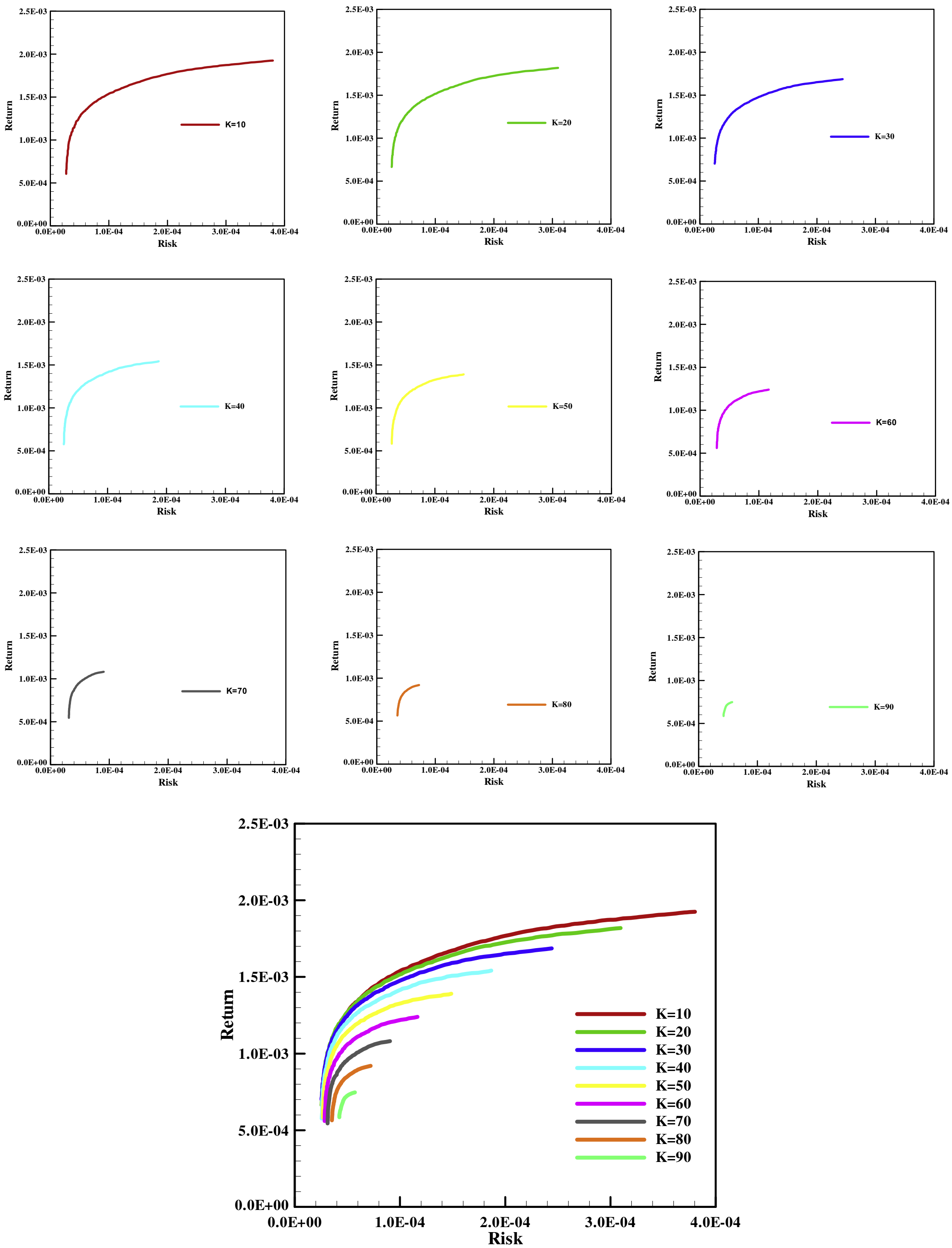

Fig. 2. CCEF of mean–variance model for FTSE data.

Note that although they are plotted in return and risk coordinates, each figure actually represents results of CCEF in different data sets or risk measures. We illustrate the computation results made from above GA for three data sets in mean–variance model first, and then discuss the S&P 100 data in three different risk measures.

In Figs. 1–3 we show these comparisons for mean–variance model, where CCEF resulted from the nine different K values are arranged by 1–3 rows individually and CCEF of mean–variance model including $K = 10$–90 is arranged by fourth row. These three groups of data under evaluation reach similar results in mean–variance model, at least from our macroscopic point of view; these three figures make a comparison based on different K values. As a result, CCEF becomes shorter following increase of K value. Three groups of data all indicate that CCEF with a K value above one third of total is obviously dominated by those with relatively less K values, which means that these K values should not be considered by investors.

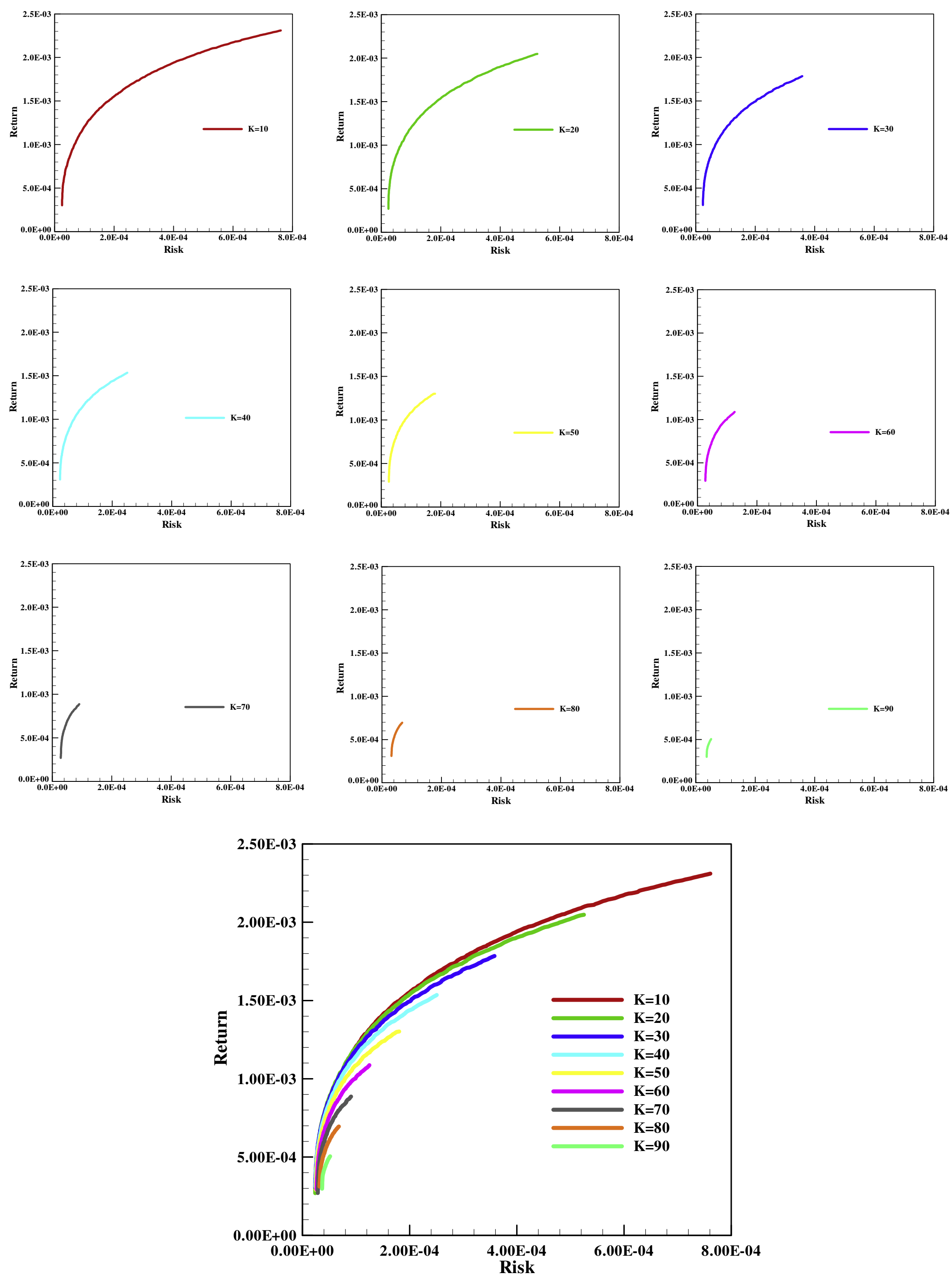

Fig. 3. CCEF of mean–variance model for S&P 100 data.

Fig. 4 is CCEF of S&P 100 data in semi-variance model, Fig. 5 is CCEF of S&P 100 data in mean absolute deviation model, and Fig. 6 is CCEF of S&P 100 data in variance with skewness model. All indicate that CCEF for the different risk models have similar results as those in mean–variance model. This further proves that the proposed GA can provide a consistent and reliable result for different market or risk preference. We also learned from previous discussion that the different risk models in CCEF cannot be directly compared since the manner of risk measures is different.

In Fig. 6, we choose the coefficient of skewness θ as 0.001, which represents further information of data. Note that in our variance with skewness model the trade-off is among mean, variance and skewness. Therefore value of θ might not have same influence to different data set. In practice, investors should more concern about how to decide the value.

Fig. 7 shows the CPU time (s) for various values of K with different risk measures in S&P 100 data. The computation time increases almost linearly as we increase K value. In spite of consuming time, Investors do not need to waste time to compute those high K

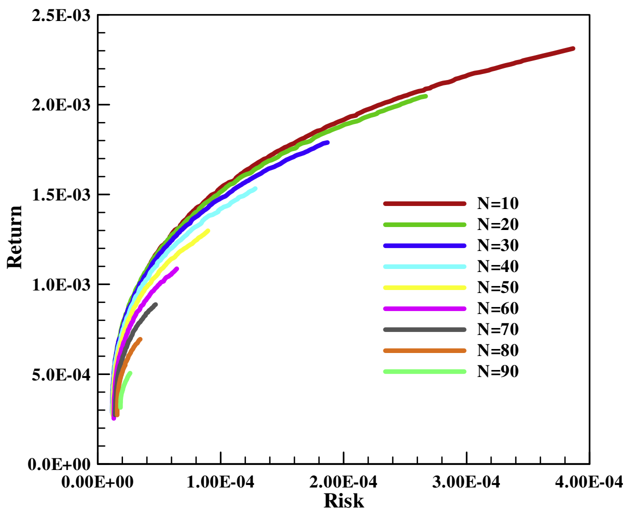

Fig. 4. CCEF of semi-variance model for S&P 100 data.

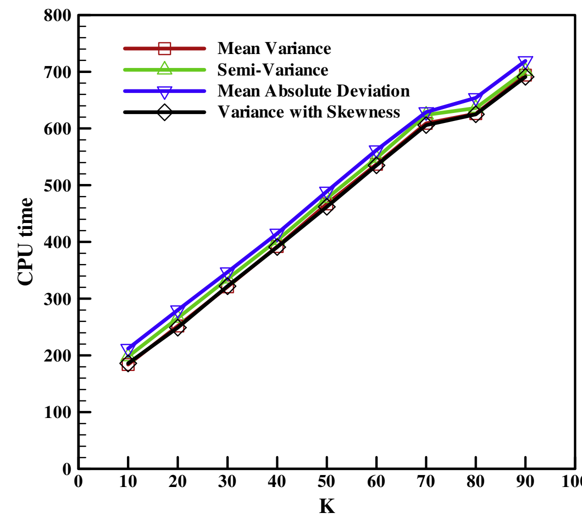

Fig. 7. CPU time (s) for various values of K in different risk models.

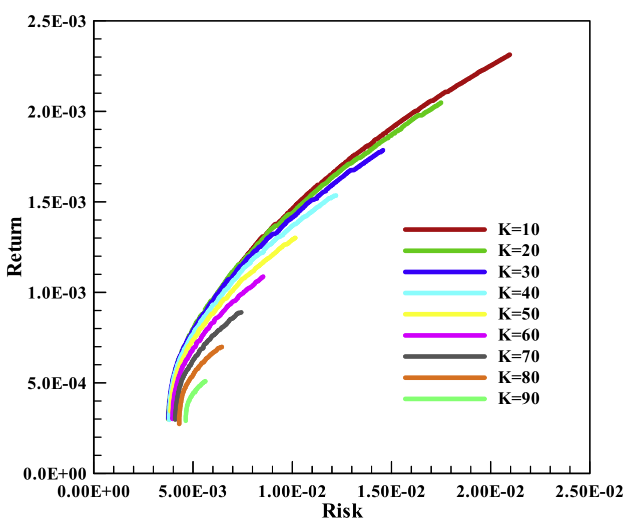

Fig. 5. CCEF of mean absolute deviation model for S&P 100 data.

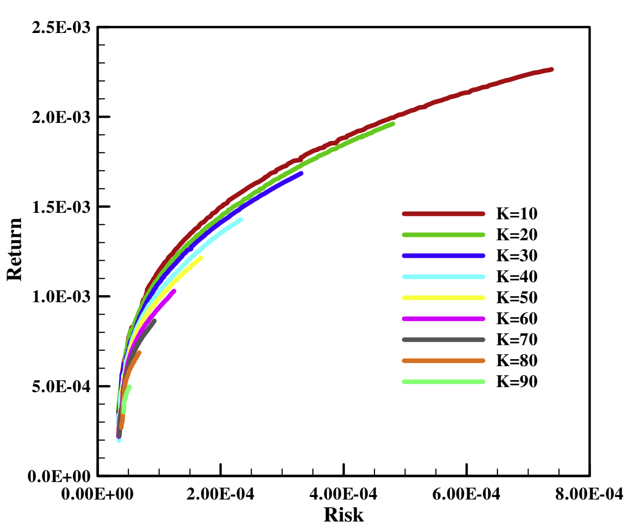

Fig. 6. CCEF of variance with skewness model for S&P 100 data.

values since they are dominated by lower one as mention above. Therefore, our GA seems to be able to efficiently obtain a near-optimal solution without difficulty, especially when the mean–variance, semi-variance and variance with skewness measures are employed as the objective function.

In this paper we do not compared GA with other mathematical programming algorithms in efficiency because we do emphasize that portfolio optimization problems can be solved by genetic algorithm for different measures of risk or risk preference. In other words, when we cannot solve the portfolio optimization problems analytically in complex cases for specific risk measures, the proposed GA is a good alternative.

6. Conclusions

Genetic algorithm is robust to solve mixed nonlinear and integer programming problems and effective for solving the portfolio optimization problems in different risk measures. GA prominent advantage over other exact search methods is its flexibility and its ability to easily obtain a good solution to a problem where the other deterministic methods cannot achieve optimality in an easy manner. The main objective of this paper was to investigate genetic algorithm for solving difficult portfolio optimization problems with different risk models. Specifically, a number of portfolio optimization problems including cardinality constraint can be solved by the state-of-the-art GA in a practical amount of time if we use mean–variance, semi-variance and variance with skewness as the measures of risk. The application of our GA in the proposed portfolio optimization problem is attractive because they are able to deal with a class of objective functions which are difficult to solve by other exact search algorithms found in literature.

The contribution of this paper showed the efficiency using GA to solve these portfolio optimization problems in different risk measures. It also verified that investors should not consider K values above one third of total assets since they are obviously dominated by those with relatively less K values.

The GA method developed in the paper can provide an efficient and convenient tool for investors. With different risk tendencies, investors are able to find efficient frontier based on a fixed amount of assets, as well as a lower bound of each asset to avoid minor investment which might increase transaction costs. In terms of number of assets hold in the portfolio, our research provides a clear fact that a small size of portfolio could have a better performance than those of bigger one. The future study is to extend a class of problems approach such as:

(a) Investigating other algorithms and their performance in different risk measures, such as simulated annealing, tabu search and neural network.
(b) Applying other algorithms to portfolio optimization problems and comparing the results to those obtained with previous heuristic and mathematical programming algorithms.
(c) Analyzing other algorithms that are more appropriate and efficient for specific risk measures.

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