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A hybrid differential evolution algorithm to vehicle routing problem with fuzzy demands

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In this paper, the vehicle routing problem with fuzzy demands (VRPFD) is considered, and a fuzzy chance constrained program model is designed, based on fuzzy credibility theory. Then stochastic simulation and differential evolution algorithm are integrated to design a hybrid intelligent algorithm to solve the fuzzy chance constrained program model. Moreover, the influence of the dispatcher preference index on the final objective of the problem is discussed using stochastic simulation, and the best value of the dispatcher preference index is obtained.

1. Introduction

The vehicle routing problem (VRP) is concerned with finding efficient routes of minimum total cost, beginning and ending at a central depot, for a fleet of vehicles to serve a number of customers for some commodity, such that each customer is visited exactly once by one vehicle, and satisfying some side constraints such as capacity constraints, duration constraints and time window constraints. This problem was first introduced in [1]. Hundreds of papers in world literature have been devoted to this problem. But most of them assume that all information is deterministic, such as customer information, vehicle information, state of roads information as well as dispatcher information and so on, and the proposed algorithm is only used to solve the deterministic VRP.

Actually, in some new systems, it is hard to describe the parameters of the vehicle routing problem as deterministic VRP because there exists much uncertain data such as customer demands, traveling time as well as the set of customers to be visited. We call these problems non-deterministic VRP. In this respect, stochastic vehicle routing problem (SVRP) and fuzzy vehicle routing problem (FVRP) are the main research objects. SVRP arises whenever some elements of the problem are random. Common examples are stochastic demands and stochastic travel times. Sometimes, the set of customers to be visited is not known with certainty. In such a case, each customer has a probability of being present. Lots of researcher gave many models and algorithms for SVRP [2–5]. FVRP arises whenever some elements of the problem are uncertain, subjective, ambiguous and vague [6]. For instance, in our problem, the information about demand at each customer is often not precise enough. For example, based on experience, it can be concluded that demand of a customer is “around 50 units”, “between 20 and 60 units”, etc. Generally, we can use fuzzy variables to deal with these uncertain parameters, as was first presented in [6] in VRP, and Cheng and Gen [7] used a genetic algorithm to solve the vehicle routing problem with fuzzy due-time. Moreover, Lai et al. [8] modeled VRP with fuzzy travel times by fuzzy programming with a possibility measurement, and adopted the genetic algorithm to solve the model. Zheng and Liu [9] researched the vehicle routing problem with fuzzy

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travel time, and presented a chance constrained program (CCP) model with credibility measurement, then integrated a fuzzy simulation and genetic algorithm (GA) to design a hybrid intelligent algorithm to solve the model. The FVRP differs from their deterministic counterpart in several fundamental respects. The concept of a solution is different, several fundamental properties of deterministic VRP no longer hold in FVRP, and solution methodologies are considerably more intricate.

Recently, differential evolution (DE), as a novel evolutionary technique has been developed, and it was originally proposed for continuous optimization [10,11]. DE is a population-based globally evolutionary algorithm, which uses a simple operator to create new candidate solutions and a one-to-one competition scheme to select new candidates greedily. Due to its simple structure, easy implementation, quick convergence, and robustness, DE has been turned out to be one of the best evolutionary algorithms for solving continuous problems in a variety of fields. Nevertheless, due to DEs continuous nature, the research work on DE for combinatorial optimization is very limited. Obviously, it is difficult to apply DE to different areas of problems other than continuous optimization that inventors originally focused on. Recently, some researchers have used DE to design a machine layout problem [10,12], and to solve manufacturing problems with mixed integer discrete variables [13]. But to the best of our knowledge, no work can be found about VRP that uses differential evolution. In this paper, we will firstly adopt a differential evolution algorithm to solve vehicle routing problem with fuzzy demand, and the proposed differential evolution algorithm can also solve other deterministic counterpart in that the VRPFD considers more complicated side constraints.

This paper is organized as follows: In Section 2, we give some basic concepts on fuzzy theory. In Section 3, we introduce vehicle routing problem with fuzzy demand and present a CCP model, where we will measure fuzzy events with credibility. Then we integrate stochastic simulation and a differential evolution algorithm to design a hybrid intelligent algorithm to solve this model in Section 4. In Section 5, we will give two experiments to reveal the effectiveness of the hybrid intelligent algorithm. In the final section, we summarize the contributions of the paper.

2. Fuzzy credibility measure theory

The concept of the fuzzy set was initiated in [14] via the membership function. And then it has been well developed and applied in a wide variety of real problems. In order to measure a fuzzy event, the term fuzzy variable was introduced in [15], and possibility measure theory of fuzzy variable was proposed in [16] Recently, credibility theory was founded by Liu [17].

In this section, we briefly introduce some basic concepts and results about fuzzy measure theory. First, we will introduce the axioms system of possibility measure theory [17], which forms the basis of credibility measure theory. Let Θ be a nonempty set, and P the power set of Θ. Each element in P is called an event, and ϕ is an empty set. In order to present an axiomatic definition of possibility, it is necessary to assign a number $\mathrm{Pos}\{A\}$ to each event A, which indicates the possibility that A will occur. In order to ensure that the number $\mathrm{Pos}\{A\}$ has certain mathematical properties which we intuitively expect, we accept the following four axioms:

Axiom 2.1 (*[17]*). $\mathrm{Pos}\{\Theta\} = 1$;

Axiom 2.2. $\mathrm{Pos}\{\phi\} = 0$;

Axiom 2.3. For each $A_i \in p(\Theta)$, $\mathrm{Pos}\{\cup_i A_i\} = \sup_i \mathrm{Pos}\{A_i\}$;

Axiom 2.4. If Θ_i is a non-empty set, and the set function $\mathrm{Pos}_i\{\}$, $i = 1, 2, \ldots, n$, satisfies above three axioms, and $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$, then for each $A \in p(\Theta)$, $\mathrm{Pos}\{A\} = \sup_{(\theta_1,\theta_2,\ldots,\theta_n)\in A} \mathrm{Pos}_1\{\theta_1\} \wedge \mathrm{Pos}_2\{\theta_2\} \wedge \cdots \mathrm{Pos}_n\{\theta_n\}$.

The above four axioms form the basis of credibility measure theory, we can obtain all concepts of credibility theory from them [17].

Definition 2.5 (*[17]*). Let $(\Theta, P(\Theta), \mathrm{Pos})$ be a possibility space, and A be a set in $p(\Theta)$, then the necessity measure of A is defined by $\mathrm{Nec}\{A\} = 1 - \mathrm{Pos}\{A^c\}$.

Definition 2.6 (*[17]*). Let $(\Theta, P(\Theta), \mathrm{Pos})$ be a possibility space, and A be a set in $p(\Theta)$, then the credibility measure of A is defined by $Cr\{A\} = \frac{1}{2}(\mathrm{Pos}\{A\} + \mathrm{Nes}\{A\})$.

Obviously, a fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is 0. However, the fuzzy event must hold if its credibility is 1, and fail if its credibility is 0 [17]. The credibility measure is self dual — in the theory of fuzzy subsets the law of credibility plays a role similar to that played by the law of probability in measurement theory for ordinary sets.

Now let us consider a triangular fuzzy variable $d = (d_1, d_2, d_3)$, d is described by its left boundary d_1, and its right boundary d_3. Thus, the dispatcher or analyst studying the problem can subjectively estimate, based on his experience and intuition and/or available data, the demand of the customer will not be less than d_1 or greater than d_3. The value of d_2

corresponding to a grade of membership of 1 can also be determined by a subjective estimate. From the definitions of possibility, necessity and credibility, it is easy to obtain:

$$\mathrm{Pos}\{d \geq r\} = \begin{cases} 1, & \text{if } r \leq d_2 \\ \dfrac{d_3 - r}{d_3 - d_2}, & \text{if } d_2 \leq r \leq d_3 \\ 0, & \text{if } r \geq d_3 \end{cases} \tag{1}$$

$$\mathrm{Nec}\{d \geq r\} = \begin{cases} 1, & \text{if } r \leq d_1 \\ \dfrac{d_2 - r}{d_2 - d_1}, & \text{if } d_1 \leq r \leq d_2 \\ 0, & \text{if } r \geq d_2, \end{cases} \tag{2}$$

$$Cr\{d \geq r\} = \begin{cases} 1, & \text{if } r \leq d_1 \\ \dfrac{2d_2 - d_1 - r}{2(d_2 - d_1)}, & \text{if } d_1 \leq r \leq d_2 \\ \dfrac{d_3 - r}{2(d_3 - d_2)}, & \text{if } d_2 \leq r \leq d_3 \\ 0, & \text{if } r \geq d_3. \end{cases} \tag{3}$$

3. The fuzzy chance constrained program model of VRPFD

We assume that: (a) each vehicle has a container with a physical limitation and total loading of each vehicle cannot exceed its capacity C; (b) each vehicle has maximum distance constraints, and total travel distance of each vehicle cannot exceed L; (c) a vehicle will be assigned for only one route on which there may be more than one customer; (d) a customer will be visited by one and only one vehicle; (e) each route begins and ends at the company depot (0); (f) the demand of each customer is a triangular fuzzy number $d = (d_1, d_2, d_3)$, the distance between customer i and customer j is c_{ij}, and there are $\overline{k}$ vehicle in the depot. For convenience, the capacity of the vehicles is the same, and the maximum distance that each vehicle can travel is equal — they can be denoted respectively as C and L.

We assume that service is provided by vehicles of the same capacity. We will denote vehicle capacity by C and the fuzzy number representing demand at the ith customer by $d_i = (d_{1i}, d_{2i}, d_{3i})$. After serving the first k customers, the available capacity of the vehicle will equal $Q_k = C - \sum_{i=1}^{k} d_i$, Q_k is also a triangular fuzzy number by using the rules of fuzzy arithmetic, and

$$Q_k = \left(C - \sum_{i=1}^{k} d_{3i}, C - \sum_{i=1}^{k} d_{2i}, C - \sum_{i=1}^{k} d_{1i} \right) = (q_{1,k}, q_{2,k}, q_{3,k}).$$

We obtain the credibility that the next customer demand does not exceed the remaining capacity of the vehicle, i.e.

$$\begin{aligned} Cr &= Cr\{d_{k+1} \leq Q_k\} = Cr\{(d_{1,k+1} - q_{3,k}, d_{2,k+1} - q_{2,k}, d_{3,k+1} - q_{1,k},) \leq 0\} \\ &= \begin{cases} 0, & \text{if } d_{1,k+1} \geq q_{3,k} \\ \dfrac{q_{3,k} - d_{1,k+1}}{2 * (q_{3,k} - d_{1,k+1} + d_{2,k+1} - q_{2,k})}, & \text{if } d_{1,k+1} \leq q_{3,k}, d_{2,k+1} \geq q_{2,k} \\ \dfrac{d_{3,k+1} - q_{1,k} - 2 * (d_{2,k+1} - q_{2,k})}{2 * (q_{2,k} - d_{2,k+1} + d_{3,k+1} - q_{1,k})}, & \text{if } d_{2,k+1} \leq q_{2,k}, d_{3,k+1} \geq q_{1,k} \\ 1, & \text{if } d_{3,k+1} \leq q_{1,k}. \end{cases} \end{aligned} \tag{4}$$

As we now know, if the vehicle's remaining capacity is greater and the demand at the next customer is less, then the vehicle's "chance" of being able to finish the next customer's service become greater. That is to say, the greater the difference between available capacity of the vehicle and demand at the next customer, the greater our preference to send the vehicle to serve the next customer. We will describe the preference index by Cr, which denotes the magnitude of our preference to send the vehicle to the next customer after it served current customer in according to formulation (4). Obviously, $Cr \in [0, 1]$. When $Cr = 0$, we are completely sure that the vehicle should return to the depot. When $Cr = 1$, we are absolutely certain that we want the vehicle to serve the next customer.

Let the dispatcher preference index equal Cr^*, $Cr^* \in [0, 1]$. And Cr^* expresses the dispatcher's attitude toward risk. When the dispatcher is a risk lover, he will choose lower values of parameter Cr^*, which indicate the dispatcher endeavor to use the vehicle available capacity as much as possible, although there is a rise in the number of cases in which the vehicle arrives at the next customer and is not able to carry out planned service due to small available capacity. On the other hand, if the dispatcher chooses the greater Cr^* in order to ensure that the chance between the available capacity of vehicle greater and the demand of next customers is greater, it indicates the dispatcher is risk averse.

So, according to the dispatcher preference index value and the credibility that the next customer demand does not exceed the remaining capacity of the vehicle, a decision must be made as to whether to send it to the next customer or return it to depot. In this paper, the decision was made as follows: if the relation $Cr \geq Cr^*$ is fulfilled, then the vehicle should be sent to the next customer; otherwise, the vehicle should be returned to the depot, and send another vehicle to the next customer. We do not terminate the above process until all of the customers' demands are fulfilled.

Moreover, the vehicle routes are designed in advance by applying the proposed algorithm. But the actual value of demand of a customer is only known when the vehicle reaches the customer. Due to the uncertainty of demand at the customers, a vehicle might not be able to service a customer once it arrives there due to insufficient capacity when the vehicle implements the planned route. In this paper, it is assumed in such situations the vehicle returns to the depot, empties what it has picked up thus far, returns to the customer where it had a "failure" and continues service along the rest of the planned route, accordingly — there arises additional distance due to route failure.

So, we must consider the additional distance that the vehicle makes due to "failure" arising at some customers along the route when evaluating the planned route. Parameter Cr^* which is subjectively determined has an extremely great impact on both the total length of the planned routes and on the additional distance covered by vehicles due to "failures" at some customers. As already mentioned, lower values of parameter Cr^* express the dispatcher's desire to use vehicle capacity the best he can. These values result in shorter planned distances. But lower values of parameter Cr^* increase the number of situations in which vehicles arrive at a customer and are unable to service them, thereby increasing the total distance they cover due to the "failure". We use stochastic simulation to evaluate the additional distance due to route failure in the following section. On the other hand, higher values of parameter Cr^* are characterized by less utilization of vehicle capacity along the planned routes and less additional distance to cover due to failures. The problem logically arises of determining the value of parameter Cr^* which will result in the least total sum of planned route lengths and additional distance covered by vehicles due to failure [6].

We assume the decision variable $x_{ijk} = 1$, if arc (i, j) belongs to the route operated by vehicle k, otherwise is 0; $y_{ik} = 1$, if customer i is serviced by vehicle k, otherwise is 0.

The corresponding chance constrained program (CCP) mathematical formulation of VRPFD based on credibility theory is given by:

$$\min \quad \sum_{k=1}^{\bar{k}} \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} x_{ijk} \tag{5}$$

$$\min \quad c' \tag{6}$$

$$\text{s.t.} \quad Cr\left(\sum_{i=1}^{n} d_i y_{ik} \leq C\right) \geq Cr^*, \quad k = 0, 1, \ldots, \bar{k} \tag{7}$$

$$\sum_{i=0}^{n} \sum_{k=1}^{\bar{k}} x_{ijk} = 1, \quad j = 1, 2, \ldots, n; \tag{8}$$

$$\sum_{i=0}^{n} x_{ijk} - \sum_{i=0}^{n} x_{jik} = 0, \quad j = 0, 1, \ldots, n; k = 0, 1, \ldots, \bar{k}; \tag{9}$$

$$\sum_{j=1}^{n} x_{0jk} \leq 1, \quad k = 1, 2, \ldots, \bar{k}; \tag{10}$$

$$\sum_{i=0}^{n} x_{ijk} = y_{jk}, \quad j = 0, 1, \ldots, n; k = 0, 1, \ldots, \bar{k}; \tag{11}$$

$$\sum_{j=0}^{n} x_{ijk} = y_{ik}, \quad i = 0, 1, \ldots, n; k = 0, 1, \ldots, \bar{k}; \tag{12}$$

$$\sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} x_{ijk} \leq L \quad k = 0, 1, 2, \ldots, \bar{k} \tag{13}$$

$$x_{ijk} \in \{0, 1\}, \qquad y_{ik} \in \{0, 1\}, \quad i, j = 0, 1, \ldots, n; k = 0, 1, \ldots, \bar{k}. \tag{14}$$

The objective function (5) seeks to minimize total planned travel distance. The objective function (6) seeks to minimize total additional travel distance due to routes failure, c' can be obtain by stochastic simulation algorithm in Section 4.1. The sum of the planned distance and additional distance can be obtained by the improved differential evolution algorithm in Section 4.2. Chance constraint (7) assures that all customers are visited within vehicle capacity with a confidence level. Constraints (8) ensure that each customer is visited by exactly one vehicle. Constraints (9) guarantee that the same vehicle arrives and departs from each customer it serves. Restrictions (10) define that at most $\bar{k}$ vehicles are used. Restrictions (11) and (12) express the relation between two decision variable. Restrictions (13) are the maximum distance constraints, L is the upper limit on the total distances transported by a vehicle in any given section of the route. Finally, constraints (14) define the nature of the decision variable.

4. Hybrid intelligent algorithm

In this paper, we design a hybrid intelligent algorithm integrating stochastic simulation and a differential evolution algorithm to solve the above fuzzy chance constrained program model. For a given value of dispatcher preference index Cr^*, we adopt a triangular fuzzy number within vehicle capacity to represent demand of each customer, but the real value of demand of a customer is a real number within fuzzy boundaries when the vehicle reaches the customer. Firstly, we apply the method of stochastic simulation to simulate the real value of demands at each customer in order to obtain the planned distance according to coding method in Section 4.2 and the additional distance due to "routes failure". Then we apply an improved differential evolution algorithm to obtain the total expected distance, which is the total sum of planned route lengths and the additional distance covered by vehicles. The objective is to obtain the best value of parameter Cr^* which will result in the least total sum of planned route lengths and additional distance covered by vehicles due to failure.

4.1. Stochastic simulation algorithm

Because the demand on each customer is a triangular fuzzy number, we cannot deal with it directly as a deterministic number by applying other algorithms that solve the deterministic vehicle routing problem. The real value of demand of a customer when the vehicle reaches the customer can be considered as a deterministic number by simulation. For each feasible planned route that the solution of the above model stands for, we obtain an approximate estimate about additional distances (c') due to routes failure by a stochastic simulation algorithm. We summarize stochastic simulation as follows.

Algorithm 1. Step 1: For each customer, estimate the additional distances by simulating "actual" demands. The "actual" demands were generated by following processes: (1) randomly generate a real number x in the interval between the left and right boundaries of the triangular fuzzy number representing demand at the customer, and compute its membership u; (2) generate a random number a, $a \in [0, 1]$; (3) compare a with u, if $a \leq u$, then "actual" demand at the customer is adopted as being equal to x;in the opposite case, if $a > u$, it is not accepted that demand at the customer equals x. In this case, random numbers x and a are generated again and again until random number x and a are found that satisfy relation $a \leq u$; (4) check and repeat (1)–(3), and terminate the process when each customer has a simulation "actual" demand quantity.

Step 2: Move along the route designed by credibility theory and accumulate the amounts picked up from each customer, and calculate the additional distance due to routes failure in terms of the "actual" demand.

Step 3: Repeat Step 1 and Step 2 M times.

Step 4: Compute the average value of additional distance by M times simulation, and it is regarded as the additional distance.

4.2. Differential evolution algorithm

Differential evolution algorithm (DE) grew out of Ken Price's attempts to solve the Chebychev polynomial fitting problem that had been posed to him by Storn and Price [10]. DE combines simple arithmetic operators with the classical operators of crossover, mutation and selection to evolve from a randomly generated starting population to a final solution. The crucial idea behind DE is a new scheme for generating trial parameter vectors. DE generates new parameter vectors by adding the weighted difference vector between two population members to a third member. If the resulting vector yields a lower objective function value than a predetermined population member, the newly generated vector replaces the vector with which it was compared. In addition, the best parameter vector is evaluated for every generation in order to keep track of the progress that is made during the minimization process. DE uses a rather greedy and less stochastic approach to problem solving compared to evolution algorithms. These simple, yet powerful and straightforward, features make it very attractive in numerical optimization.

When applying DE to solve VRP, we face many difficulties. How to deal with the fuzzy number about customer demand and to extend the canonical DE to optimize integer variables problems are the key problems. The customer permutation-based encoding scheme [1,9,18] has been widely used for VRP. However, due to DEs continuous nature, the standard encoding scheme of DE cannot be directly adopted for VRP. So, the important issue in applying DE to VRP is to find a suitable mapping between customer sequence and individuals (continuous vectors) in DE. Formally, according to the canonical DE strategy DE/rand/1/bin, the improved differential evolution algorithm is briefly described in order to solve VRPFD effectively, as follows. And more detailed working strategies about DEs can be found in [10,11,21].

4.2.1. Coding and initiation population

As with all evolutionary optimization algorithms, DE is a novel parallel direct search method which works with a population of solutions, not with a single solution for the optimization problem. Like conventional GA for the VRP [18, 9], a chromosome Chrom$(i, :)$ is a sequence (permutation) of n customers. We adopt this ordinal number encoding method, the initial population is chosen randomly and population size is NP, NP doesn't change during the minimization process, and each individual is an n-dimensional solution vectors that is a permutation about customer number, i.e. $\text{Chrom}(i, :) = \text{randperm}(n)$, $i = 1, 2, \ldots, NP$, where n is the number of the customers. We check the capacity chance constraint (7) and distance constraint (13) at the same time from the first gene of chromosome, if it does not violate the constraints, considering the next gene; if it violates the constraints in some gene, we consider using other vehicle from this gene, starting, and repeating the above process, until all of customers have been serviced. That is to say, the demand of each customer must satisfy the capacity chance constraint in a route, and in any route the total traveled distance cannot exceed the maximum distance of the associated vehicle, For instance, if there are 10 customers, a randomly generated chromosome is 1 3 6 8 9 5 4 10 2 7, 1 3 6 8 9 5 4 10 2 7, can be interpreted as $r = 3$ feasible routes: 0–1–3–6–0, 0–8–9–5–0, and 0–4–10–2–7–0. If $\bar{k} >= r$, then this chromosome is legal; otherwise, it is illegal.

In order to prevent an illegal chromosome entering the next generation in great probability, a penalty function is designed. R is the total distance of vehicles traveled for the corresponding chromosome, and let $m = r - \bar{k}$. If $r > \bar{k}$, then $m > 0$, and $R = R + Z \times m$, where Z is a very large integer. If $r < \bar{k}$, then $m = 0$. The fitness function can be expressed as $f = 1/(R + Z \times m)$.

We summarize the above process as fellows.

Algorithm 2. Step 1: Randomly generate a chromosome of customer arrangements.

Step 2: Choosing the first customer of chromosome, according to the customer demand and the vehicle remaining capacity, we can obtain Cr from formulation (4). For a dispatcher preference index value Cr^*, if $Cr \geq Cr^*$, then the customer is assigned to the current vehicle; otherwise use another vehicle to service this customer.

Step 3: Delete the first customer from the chromosome.

Step 4: Repeat step 2, step 3, if all of the customers have been assigned to routes, we obtained a feasible chromosome.

Step 5: Repeat the first to the forth steps for a given population size NP.

4.2.2. Mutation operation

The chromosome of offspring is generated by parent gene difference, and mutation is an operation that adds a vector differential to a population vector of individuals, according to the following equation:

$$v(i, :) = \text{Chrom}(c, :) + F * [\text{Chrom}(a, :) - \text{Chrom}(b, :)] \tag{15}$$

where $a, b, c, i \in [1, NP]$ randomly selected, they are mutually different from each other and also different from the running index i, i.e. $a \neq b \neq c \neq i$, they refer to three randomly chosen vectors of population. The scaling factor $F \in [0, 2]$ is a real constant, it controls amplification of the differential variation $\text{Chrom}(a, :) - \text{Chrom}(b, :)$. Because we adopt an ordinal encoding scheme, and each chromosome represents a permutation of the customers, each gene stands for a customer, when the offspring gene lie outside their allowed ranges after performing formulation (15), we must consider an auxiliary operator based on integer order criterion (IOR) just like the largest-order-value (LOV) rules that Qian et al. [19] proposed. The largest gene of offspring gives the largest customer ordinal number n, the second evaluated as $n - 1$, the rest may be deduced by analogy. We can prove that this operator equates to an affine transform [20], for example, the offspring chromosome is [−7.1, 1.3, −5.6, 2.5, −3.7, 0, 3.3, 5.4], the number of the customers is 8, we obtain the offspring chromosome that is [1, 5, 2, 6, 3, 4, 7, 8] by using IOR. We adopt the ordinal encoding method and an auxiliary operator based on IOR to improve the mutation operation, which is a crucial reason that differential evolution can solve vehicle routing problem. If we adopt other usual rounded number methods to obtain integer, we cannot solve the vehicle routing problem.

4.2.3. Crossover operation

In order to increase the potential diversity of the perturbed parameter vectors, a crossover operation is introduced after the mutation operation. Crossover operation is employed to generate a temporary or trial vector by replacing certain parameters of the target vector by the corresponding parameters of a randomly generated donor vector. The trial vector is formed by the following equation:

$$\text{trial}(i, j)^{G+1} = \begin{cases} v(i, j)^{G+1}, & \text{rand}_j \leq CR \quad \text{or} \quad j = \text{rand}n(i) \\ \text{Chrom}(i, j)^G, & \text{otherwise}, \end{cases} \tag{16}$$

where rand_j is a random value within interval [0, 1], $\text{rand}n(i)$ is a randomly chosen index from the set of customers. We obtain the ordinal valued vector after mutation operation, including auxiliary operator, therefore all vectors of formulations (16) are integer valued vectors. Obviously, the index $\text{rand}n(i)$ refers to a randomly chosen vector parameter and it is used to ensure that at least one vector parameter of each individual trial vector $\text{trial}(i, :)^{G+1}$ differs from its counterpart in the previous generation $\text{Chrom}(i, :)^G$. In other words, a certain sequence of the vector elements of $\text{trial}(i, :)^{G+1}$ is identical to the

Table 1
The relative parameters of model and algorithm.

n	C	$\bar{k}$	MAXGEN	M	L	NP	CR_{min}	CR_{max}	F
30	8	30	100	100	2000	60	0.3	0.9	0.5

elements of $v(i,:)^{G+1}$ if it yields a smaller stochastic number than CR, otherwise the other elements of $\text{trial}(i,:)^{G+1}$ acquire the original values of $\text{Chrom}(i,:)^{G}$.

In formulations (16), G is the number of current iteration, $CR \in [0, 1]$ is the crossover probability factor. In order to improve the population's diversity and the ability to break away from the local optimum, we present a new self-adapting crossover probability factor, namely the crossover probability (CR) is time varying, it changes from small to large with iteration number G. I.e.

$$CR = CR_{\min} + G * \frac{CR_{\max} - CR_{\min}}{\text{MAXGEN}}, \tag{17}$$

where CR_{min} is the proposed minimum crossover probability, and CR_{max} is the maximum crossover probability, MAXGEN is the number of maximum iteration. In the early stage of evolution, the crossover probability is smaller, which can improve the global searching capability; in the later stage of evolution, the crossover probability is larger, which can improve the local searching capability.

4.2.4. Estimation and selection operation

According to formulation (5) we obtained the total planned length of all routes (c), and obtained the additional distance covered due to routes failures (c') based on formulation (6). We defined the fitness value as a reciprocal of the total length of all routes, i.e. $f = 1/R = 1/(c + c')$. The selection scheme of DE differs from the other evolutionary algorithms. The fitness of the temporary individual $\text{trial}(i,:)^{G+1}$ is compared with that of its target individual $\text{Chrom}(i,:)^{G}$ in each generation, the one with higher fitness value will propagate the population of the next generation. The population of the next generation $\text{Chrom}(i,:)^{G+1} (i = 1, 2, \ldots, NP)$ is obtained by using the following greedily selection criterion:

$$\text{Chrom}(i,:)^{G+1} = \begin{cases} \text{trial}(i,:)^{G+1}, & f(\text{Chrom}(i,:)^{G}) < f(\text{trial}(i,:)^{G+1}) \\ \text{Chrom}(i,:)^{G}, & \text{otherwise}. \end{cases} \tag{18}$$

Usually, DEs main parameters are fewer than those of other evolutionary algorithms, and the performance of a DE algorithm depends on three parameters: the population size NP, the scaling factor F and the crossover probability CR. Practical advice on how to select control parameters NP F and CR can be found in [10,11,21].

5. Numerical experiments

Now we will give some examples to show models that we have just discussed and how the hybrid intelligent algorithm works. Two types of experimental conditions are created based on the size of problem (customers' number). We assume that there are 30 customers and one depot for a small size problem, and 100 customers and one depot for a large size problem. In each experiment, the coordinates of all customers and depot are generated randomly in $[100 \times 100]$, and the fuzzy demands of customers are generated randomly — they are triangular fuzzy numbers within vehicle capacity C. We obtain the additional distances due to routes failure by the stochastic simulation algorithm in Section 4.1, and obtain the planned distances and total distances by the improved differential evolution algorithm in Section 4.2.

The relative parameters for the small size problem are listed in Table 1. For the 100-customer problem, $n = 100$, $\bar{k} = 100$, $NP = 100$, and other parameters are identical with Table 1. We obtain the planned distances, additional distances and the total distances, and reveal the dispatcher preference index Cr^* which influence these traveling distances.

The hybrid intelligent algorithm was encoded in MATLAB 7.0. The value of dispatcher preference index Cr^* varied with the interval of 0 to 1 with a step of 0.1. The average computational results of 10 times are given in Tables 2 and 3 for the small size and the large size problem, respectively. Figs. 1 and 2 respectively show the tendencies regarding the planned distances, additional distances due to failures at the customers, and the total distances that vehicles were to cover when the dispatcher preference index varied.

In Tables 2 and 3, as dispatcher preference index Cr^* rose, a strictly rising tendency was noted in the total distances of planned routes, with a strict decrease in the additional distance that vehicles had to make due to failures at the customers. When the dispatcher preference indexes $Cr^* \leq 0.6$, the increasing quantity of the planned distances is smaller than the decreasing quantity of the additional distances, so the total distances is strictly decreasing as Cr^* increases from 0 to 0.6. However, when the dispatcher preference indexes $Cr^* > 0.6$, the increasing quantity of the planned distances is strictly larger than the decreasing quantity of the additional distances, so the total distances is almost strictly increasing as Cr^* increases from 0.6 to 1. And when the dispatcher preference value equals 0.6, the total distances covered by the vehicle reach the least.

Table 2
The average results with different Cr^* when $n = 30$.

Cr^*	Planned distances	Additional distances	Total distances
0.0	1672.3	2882.2	4554.5
0.1	2041.6	1426.8	3468.4
0.2	2161.6	1086.0	3247.6
0.3	2448.3	669.0	3117.3
0.4	2465.6	486.70	2952.3
0.5	2653.6	226.70	2880.3
0.6	2735.0	98.000	2833.0
0.7	2860.2	36.100	2896.3
0.8	3027.2	1.9000	3029.1
0.9	3048.4	0	3048.4
1.0	3063.2	0	3063.2

Table 3
The average results with different Cr^* when $n = 100$.

Cr^*	Planned distances ($\times 10^4$)	Additional distances ($\times 10^4$)	Total distances ($\times 10^4$)
0.0	0.4947	1.0258	1.5205
0.1	0.7344	0.4966	1.2309
0.2	0.7549	0.3706	1.1255
0.3	0.8009	0.2609	1.0618
0.4	0.8485	0.1695	1.0181
0.5	0.8963	0.0955	0.9918
0.6	0.9384	0.0374	0.9758
0.7	0.9925	0.0097	1.0022
0.8	1.0246	0.0009	1.0255
0.9	1.0550	0	1.0550
1.0	1.0433	0	1.0433

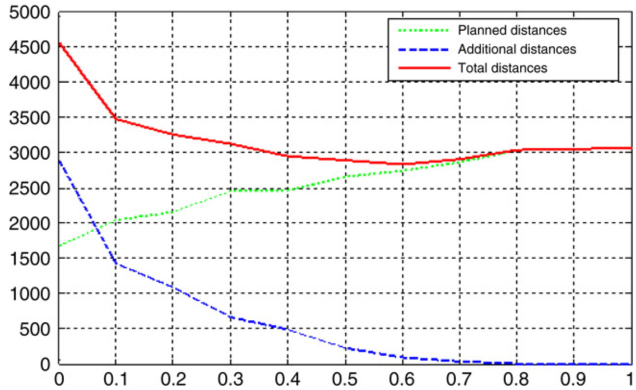

Fig. 1. The distances change tendencies with Cr^* varied when $n = 30$.

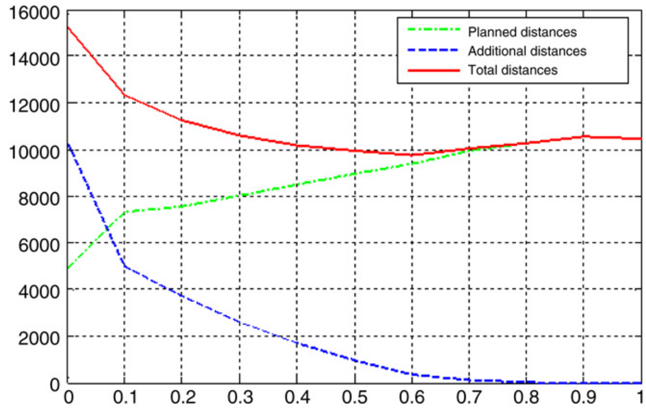

Fig. 2. The distances change tendencies with Cr^* varied when $n = 100$.

As a consequence, lower values of parameter Cr^* express our desire to use vehicle capacity the best we can. These values correspond to routes with shorter planned distances. On the other hand, lower values of parameter Cr^* increase the number of cases in which vehicles arrive at a customer and are unable to service it, thereby increasing the total additional distance they cover due to the "failure". Higher values of parameter Cr^* are characterized by less utilization of vehicle capacity along the planned routes and less additional distance to cover due to failures. Therefore, the dispatcher preference index should be 0.6 approximate.

6. Conclusion

This paper contributed to the vehicle routing problem with fuzzy demands in the following respects: (a) a chance constrained program mathematic model of VRPFD was proposed based on credibility theory; (b) stochastic simulation and differential evolution algorithms were integrated to design a hybrid intelligent algorithm to solve this problem, focusing on total traveled distance minimization; (c) the dispatcher preference index greatly influenced the length of the planned routes and the additional distances covered by vehicles due to failures at the customers, and the "best" value of parameter Cr^* was obtained by the proposed hybrid algorithm; (d) the effectiveness of the hybrid intelligent algorithm was shown by some numerical examples.

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